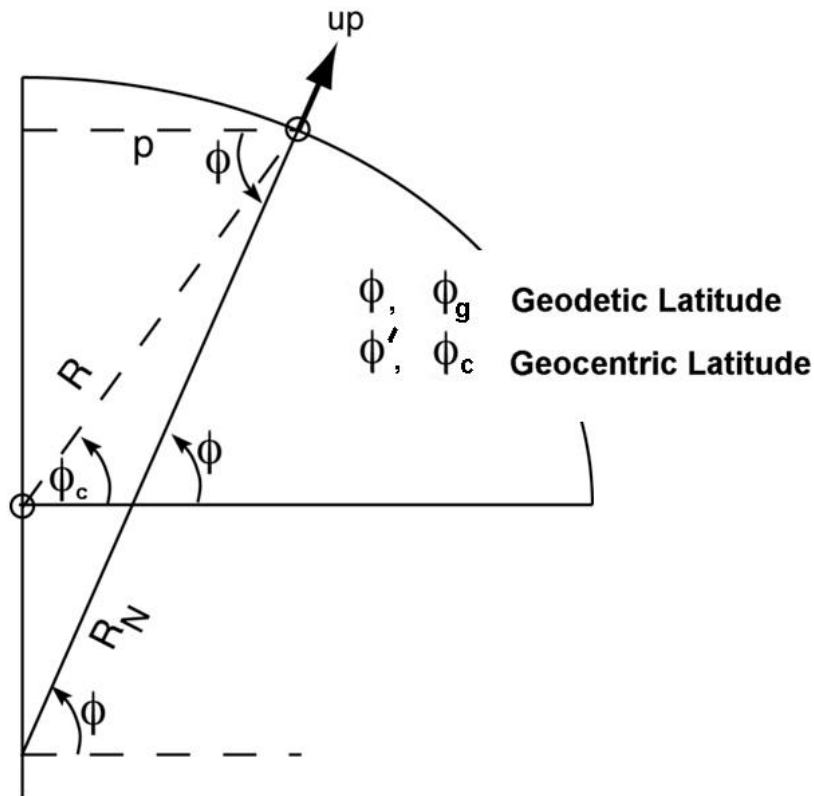


Geodetic Coordinate Conversions

James R. Clynch
 Naval Postgraduate School, 2002

I. Geodetic to/from Geocentric Latitude



Geodetic and Geocentric Latitude

A. Geodetic Latitude (ϕ , or ϕ_g) to Geocentric Latitude (ϕ' , or ϕ_c)

There are many equations that can be used. One of the most common involves the tangent of the latitude. At a geodetic or ellipsoidal height h ,

$$\tan \phi_c = \left[1 - e^2 \frac{R_N}{R_N + h} \right] \tan \phi$$

where the radius of curvature in the prime vertical, R_N , is given by

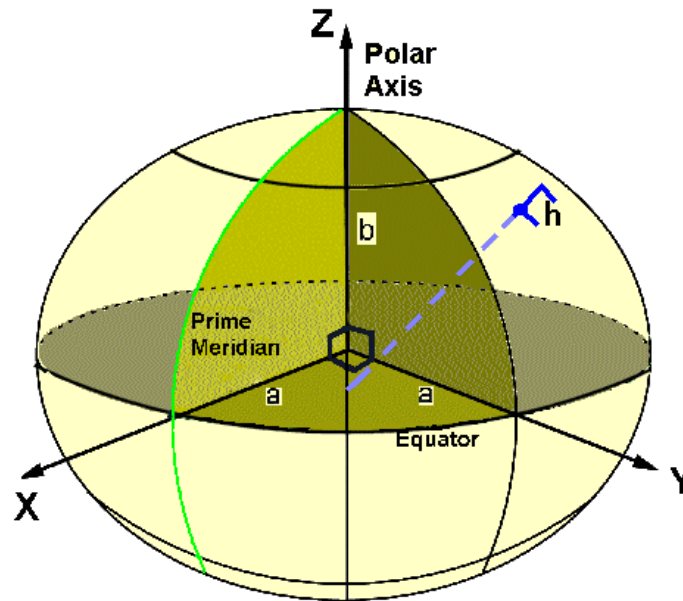
$$R_N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$

B. Geocentric Latitude (ϕ' , or ϕ_c) to Geodetic Latitude (ϕ , or ϕ_g)

This case uses the same equations. The value of R_N is found using the geocentric latitude. The error in this approximation is second order in smallness and is usually ignored. The ratio factor is, of course divided into the right hand side of the tangent equation in this case

$$\tan \phi = \left[1 - e^2 \frac{R_N}{R_N + h} \right]^{-1} \tan \phi_c$$

II. Latitude, Longitude and Height to/from ECEF (x,y,z)



Latitude, Longitude, Height To/From (X,Y,Z)

A. Latitude, Longitude, Height to ECEF xyz

There is a closed form solution for this transformation. Given geodetic latitude, ϕ , (what you find on maps), longitude, λ , and ellipsoidal height H , then

$$\begin{aligned}x &= (R_N + h) \cos \phi \cos \lambda \\y &= (R_N + h) \cos \phi \sin \lambda \\z &= ([1 - e^2] R_N + h) \sin \phi\end{aligned}$$

Two notes are important. First the longitude will be East Longitude. This is the convention for geodesy. Second notice that the ellipsoidal height, h , is used. Classical surveying gives orthometric, or mean sea level height, called H . Orthometric height is given on maps. Thus the height readily available is not the required value. The difference between ellipsoidal and orthometric height is the undulation of the vertical, a value determined by the real gravity field of the earth. This is sometimes called geoid height. It varies over values of +/- 100 m.

B. ECEF xyz to Latitude, Longitude, Height

There is no closed form solution for this transformation if the altitude is not zero. The problem is that the radius R_N is needed to find geodetic height h and geodetic latitude is needed to find R_N . The usual procedure is to iterate beginning with the assumption that there is no difference between geodetic and geocentric latitude.

First compute the longitude, which can be precisely done.

$$\begin{aligned}\lambda &= a \tan(y/x) \\ &= a \tan 2(y, x)\end{aligned}$$

The second form is the usual computer call for a 4-quadrant arctangent. Note that computer code usually returns angles in radians. These must be converted to degrees. Note also that this procedure produces East Longitude.

Next the physical radius of the point and the radius in the x-y plane are computed and used in an initial estimate of the altitude.

$$\begin{aligned}r &= \sqrt{x^2 + y^2 + z^2} \\ p &= \sqrt{x^2 + y^2}\end{aligned}$$

The geocentric latitude is computed exactly, and used as the initial value for the geodetic latitude in the iteration loop.

$$\begin{aligned}\phi_c &= a \tan(p/z) \\ &= a \tan 2(p, z) \\ \phi_{\text{now}} &= \phi_c\end{aligned}$$

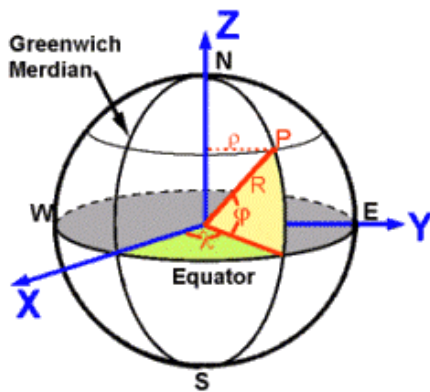
The loop is:

$$\begin{aligned}h &= \frac{p}{\cos \phi_{\text{now}}} - R_N(\phi_{\text{now}}) \\ \phi_{\text{next}} &= a \tan \left[\frac{z}{p} \left(1 - e^2 \frac{R_N}{R_N + h} \right)^{-1} \right] .\end{aligned}$$

This converges in a few iterations (4 at most) to a few centimeters. This is for positions even at earth satellite altitudes. After the geodetic latitude, ϕ , is found the ellipsoidal height, h is obtained from

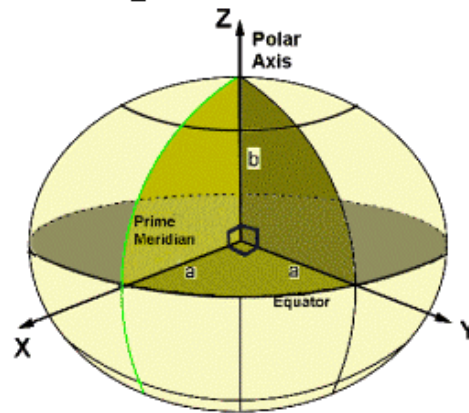
$$h = \frac{p}{\cos \phi} - R_N .$$

Cartesian to Angular



$$\begin{aligned} x &= (r + h) \cos \phi \cos \lambda \\ y &= (r + h) \cos \phi \sin \lambda \\ z &= (r + h) \sin \phi \end{aligned}$$

Spherical



$$\begin{aligned} x &= (R_N + h) \cos \phi \cos \lambda \\ y &= (R_N + h) \cos \phi \sin \lambda \\ z &= [(1 - e^2) R_N + h] \sin \phi \end{aligned}$$

Ellipsoidal