Modeling of very low frequency motions during RIPEX

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abstract

Numerical computations are used to explain the presence of Very Low Frequency motions (VLF’s), with frequencies less than 0.004 Hz, in the rip current velocity signals observed during the RIPEX field experiment. Observations show that the VLF-motions are most intense within the surfzone and then quickly taper off in the offshore direction. By comparing computed VLF-intensity ($U_{\text{rms, vlf}}$) distributions in both the cross-shore and alongshore direction with observations in a qualitative sense, the most important contributions to the VLF-dynamics are established. VLF-motions at neighboring rip-channels are seen to interact in the computations, with stronger surfzone intensity for increasing bathymetric variation. The intermittent forcing by spatially varying wave-groups is essential in obtaining the correct $U_{\text{rms, vlf}}$ distribution in the cross-shore direction suggesting this is the predominant mechanism responsible for the generation of the VLF-motions observed during RIPEX. Computations also suggest that VLF-motions can occasionally propagate offshore but are mostly confined to the surfzone corresponding to surfzone eddies. A quantitative comparison shows good correspondence between model computations and measurements of $U_{\text{rms, vlf}}$ with a model skill of $O(0.7)$, with generally increased (decreased) $U_{\text{rms, vlf}}$ during mean low

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(high) water levels.

Introduction

Low-frequency gravity-driven oscillations in rip-channel flows have been observed (Sonu, 1972; Aagaard et al., 1997; Brander and Short, 2001) and are typically thought to be related to changes in mass-flux associated with incident wave groups (Munk, 1949), or infragravity waves generated by the wave groups (Sonu, 1972, MacMahan et al., 2004a). More recently, very low frequency vorticity motions (VLF’s) in rip-current flows have been observed both in the field (Smith and Largier, 1995, MacMahan et al., 2004b) and in laboratory conditions (Haller and Dalrymple, 2001). Smith and Largier [1995] suggested the VLF’s could be due to the shear instability of the longshore current, whereas Haller and Dalrymple [2001] considered the shear instability of the rip-current itself. MacMahan et al. [2004b] hypothesize that the VLF’s are surfzone eddies generated by wave groups. In reality, all contributions are potentially present, with grouped short waves incident on a rip-channeled beach generating infragravity waves and surfzone eddies with co-existing instabilities generated by the velocity shear of the mean alongshore and cross-shore flows. The vorticity-motions in the rip-current are expected to manifest themselves outside the gravity restoring region, defined by zero-mode edge wave dispersion curves, at significantly lower frequencies \( f < 0.004 \text{Hz} \) than the infragravity waves \( (0.004 \text{ Hz} < f < 0.04 \text{Hz}) \), based on both observations (MacMahan et al., 2004a and 2004b) and theoretical considerations taking into account the velocity shear of the rip-current (Haller and Dalrymple, 2001). Given the fact that VLF-motions are vortical with little surface expression compared with the low-frequency (LF) surface gravity waves, a separate treatment is warranted for VLF motions, which is presented here.

Modelling of VLF’s in rip-channel flows has been limited to synthetic geometries both for numerical (Yu and Slinn, 2003) and laboratory simulations (Chen et al., 1999,
Haller and Dalrymple, 2001, Haas et al., 2003), and have ignored the contributions of the randomness of the incident waves on the generation of VLF-motions. Yu and Slinn [2003] observed the generation of rip-current instabilities in their computations, where the instabilities originated within the rip-current (without wave-current interaction) or in the feeder channels (with wave-current interaction). Haas et al. [2003] compared their numerical computations with the experimental data obtained by Haller and Dalrymple [2001] and concluded that inclusion of mixing associated with the vertical structure of the flow (Putrevu and Svendsen, 1999) reduced the computed instability intensities significantly. Haller and Dalrymple [2001] used a linear stability analysis to predict the temporal and spatial scales of the rip-current instabilities observed during their experiment and obtained good agreement.

Here, model computations are compared with measurements of VLF-motions obtained during the RIP-current field EXperiment (RIPEX). RIPEX provides the first comprehensive data-set of rip-current VLF’s under field conditions with varying forcing conditions throughout a large number of tidal cycles. This enables an examination of the effects of wave height variations and tidal elevation on the generation of rip-current VLF’s utilizing a non-linear flow model. The numerical model describes the propagation of wave-group energy made up of the directionally spread incident sea/swell waves over a variably bathymetry. The wave-group energy is transferred to a roller prior to dissipation to simulate breaking processes. The wave-group varying wave and roller energies are used to construct the radiation stress tensors and their divergences to drive mean-, infragravity-, and VLF-motions. The model description is given by Reniers et al. [2004a], who assessed the role of stochastic wave-group forcing in the generation of rip-channels on an initially alongshore uniform embayed beach. Comparisons of model computations with observations of infragravity waves observed during RIPEX showed good correspondence throughout the experiment (Reniers et al., 2006).

Model computations are carried out with constant parameter values for roller
dissipation and wave breaking, where the appropriate values have been determined from previous calculations considering the modeling of infragravity conditions (Reniers et al., 2006). All computations presented here are performed with wave-current interaction on the wave-group time scale. The constant model constituents of turbulent eddy viscosity and bottom friction in combination with wave-current interaction were determined quantitatively by examining the model skill, based on comparisons of simulated and observed cross-shore and alongshore velocity intensity of the VLF’s and mean flow strength resulting in a Manning number of 0.015 for the friction formulation. Computations are performed to establish the potentially important forcing mechanisms of VLF-motions by considering monochromatic, bichromatic and random waves of equal mean energy incident on a rip-channelled beach for different rip-channel configurations.

**RIPEX VLF measurements**

The measurements were obtained during RIPEX in concert with the Steep Beach Experiment (SBE) at Sand City, Monterey Bay, California, USA. The bathymetry during the experiment consisted of shore-connected shoals intersected by relatively narrow rip-channels at approximately 125 m alongshore intervals (left panel of Figure 1). Although significant changes occurred within the nearshore bathymetry, the larger morphological features stayed more or less in place. For details on the morphodynamics during the experiment refer to MacMahan et al. [2005].

The instrument set-up consisted of a cross-shore array over the shoal and an alongshore array coinciding with the beginning of the rip-channels (left panel Figure 1). An example of the measured low-pass filtered \((f < 0.04 \text{ Hz})\) current velocities obtained at puv11 (right upper and middle panel of Figure 1), located in a feeder/rip channel, displays infragravity motions and slow oscillations at the VLF time scale (250 s and longer). There is no clear evidence in the surface elevation for the presence of these VLF oscillations, similar to the observations by Oltman-Shay et al. [1989] for the shear
instabilities in the longshore current, suggesting that these motions are not associated with surface gravity waves. MacMahan et al. [2004b] show that these oscillations are indeed vorticity-induced oscillations.

The intensity of the VLF motions is defined by the root-mean-square (rms) VLF-speed, $U_{rms, vlf}$:

$$U_{rms, vlf} = \sqrt{\int_{0.004Hz}^{\delta f} S_{uu}(f) + S_{vv}(f) df}$$

where the frequency resolution, $\delta f$, equals 0.00014 Hz based on a 2 hour time series, and $S_{uu}(f)$ and $S_{vv}(f)$ represent the variance density spectra of the cross-shore and alongshore velocities. This definition holds for both measurements and model computations.

In general, the observed cross-shore distribution of $U_{rms, vlf}$ has a maximum within the surfzone and then quickly tapers off in the seaward direction (upper panel of Figure 2). The corresponding alongshore distribution of $U_{rms, vlf}$ (lower panel of Figure 2) measured at the alongshore array shows that VLF’s are present both on the shoal and within the rip-channel with an elevated intensity in the proximity of the rip channel. These distributions shows that the VLF’s are mostly contained within the surfzone, and thus, the offshore propagation of vortices generated within the surfzone is limited. The observed cross-shore and alongshore distributions are used to determine which mechanisms are the most likely to contribute to the generation of VLF’s.

**Generation of VLF’s**

Potential generation mechanisms are shear instabilities in the strongly sheared rip-current flows (Haller and Dalrymple, 2001; Yu and Slinn, 2003), the forcing of slowly modulating rip-current velocities by regular wave groups made up of two intersecting wave trains (Fowler and Dalrymple, 1990), the quasi-steady circulations induced by
individual obliquely incident wave groups considered by Ryrie [1983] or a sequence of random wave groups made up of a directionally spread incident wave field considered by Reniers et al. [2004a].

The following considerations include monochromatic waves (no groups), bichromatic waves (regular groups) and directionally spread random waves (stochastic groups). The processes investigated that are potentially important in the VLF response are rip-channel configuration and eddy-dynamics (Peregrine, 1998, Buhler and Jacobson, 2001, Reniers et al., 2004a). The various processes are assessed with a numerical model to establish the dominant contributions to the VLF-response observed during the RIPEX experiment.

Wave-current interaction is known to affect the generation and behavior of rip-current instabilities (Yu and Slinn, 2003, Haas et al., 2003). To account for the presence of a current, the wave-energy balance used by Reniers et al. [2004a] is replaced with the wave action balance:

\[
\frac{\partial N}{\partial t} + \frac{\partial N}{\partial x} \left( c_g \cos(\theta) + U \right) + \frac{\partial N}{\partial y} \left( c_g \sin(\theta) + V \right) = -\frac{D}{\sigma},
\]  

(2)

where \( N \) represents the wave action, \( D \) is the dissipation of wave energy due to breaking modelled with the dissipation formulation of Roelvink [1993], and \( x, y \) are the cross-shore (positive onshore) and alongshore coordinates following the Cartesian convention. In the same manner as Reniers et al. [2004a], the mean direction with respect to the \( x \)-axis, \( \theta \), and mean group velocity, \( c_g \), are pre-computed with the HISWA-model (Holthuijsen et al., 1978), taking into account the effects of directional spreading. Next these quantities are corrected for the presence of a current that varies on the time scale of the wave groups. The cross- and alongshore wave numbers, \( k_x \) and \( k_y \), are defined as:

\[
k_x = k_{x,0} + \tilde{k}_x,
\]

(3)
where the subscript 0 represents the wave number obtained from the pre-computed refraction, and \( \tilde{k}_x \) and \( \tilde{k}_y \) represent the corrections associated with the presence of a time-varying current. A second set of equations is used to determine these corrections (Witham, 1974, Mei, 1985):

\[
\frac{\partial \tilde{k}_x}{\partial t} + \frac{\partial \omega}{\partial x} = 0, \quad (5)
\]

\[
\frac{\partial \tilde{k}_x}{\partial y} - \frac{\partial \tilde{k}_y}{\partial x} = 0, \quad (6)
\]

where the absolute angular frequency, \( \omega \), is given by:

\[
\omega = k_x u + k_y v + \sigma \quad (7)
\]

with \( u, v \) the cross-shore and alongshore velocities on the wave-group scale, and the intrinsic angular frequency, \( \sigma \), is obtained from the linear dispersion relation:

\[
\sigma = \sqrt{gk \tanh(kh)}, \quad (8)
\]

where:

\[
k = \sqrt{(k_x)^2 + (k_y)^2}. \quad (9)
\]

The wave-group velocity \( c_g \), utilized in the wave action balance eq. 2, is calculated from the intrinsic dispersion relation (eq. 8):

\[
c_g = \frac{\partial \sigma}{\partial k} = \left( \frac{1}{2} + \frac{kh}{\sinh(2kh)} \right) c \quad (10)
\]

with \( c \) the phase speed of the waves:
\[ c = \frac{\sigma}{k} \]  

(11)

and the wave angle \( \theta \), utilized in the wave action balance, eq. 2, is obtained from the cross-shore and alongshore wave number components:

\[ \theta = \tan^{-1} \left( \frac{k_y}{k_x} \right). \]  

(12)

The equations to solve for the roller energy, radiation stresses and flow velocities are identical to the equations used by Reniers et al. [2004a] with the exception of the turbulent eddy viscosity \( \nu_t \), to which a calibration coefficient, \( \alpha \), is added:

\[ \nu_t = \alpha h \left( \frac{D_r}{\rho} \right)^{\frac{1}{3}}, \]  

(13)

where \( D_r \) is the dissipation of roller energy and \( \rho \) the water density. At the offshore boundary the incoming wave action associated with the incident waves, which can vary on the wave-group scale, is prescribed in combination with a Riemann condition to absorb the outgoing long waves.

The base case considers a single rip-channel cutting through a shore connected shoal (middle panel Figure 3). The cross-shore domain is approximately 700 m with spatially varying grid spacing (\( \Delta x \)) ranging from \( O(15) \) m offshore to \( O(4) \) m grid spacing within the surfzone. The alongshore domain is 500 m with a constant alongshore grid spacing (\( \Delta y \)) of 10 m. The rip channel and shoal dimensions are similar to the RIPEX-observations (compare with left panel of Figure 1). All computations cover a duration of two hours with a time-step of 1.2 seconds. The turbulent mixing coefficient \( \alpha \) is set at 0.1 based on the results obtained by Reniers et al. [2004b] examining the vertical structure of the mean longshore and cross-shore currents obtained during the Sandy Duck field experiment. A zero normal-flux condition is used at the lateral boundaries. At the shoreline a zero normal flux is imposed at a water depth of 0.1 m.
The generation and propagation of the VLF-motions is examined by considering the vorticity, $q$:

$$q = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \quad (14)$$

**Monochromatic Wave Forcing**

The first case considered is for monochromatic waves with constant height $H = 1$ m at the offshore boundary and wave period $T = 10$ s, normally incident on a coast with a single rip-channel. In the presence of wave-current interaction, the incident waves shoal and refract on the outgoing rip current resulting in increased wave energy within the rip-channel (upper left panel of Figure 3). This increase in wave energy is consistent with the results obtained by e.g. Yu and Slinn [2003].

Given the steady wave forcing, the observed VLF-motions in the rip-current are due to the shear instability of the rip-flow (Haller and Dalrymple, 2001; Yu and Slinn, 2003). The rip-current instabilities manifest themselves mostly outside the surf zone (middle left panel of Figure 3), defined as the area shoreward of breaking waves, occurring within approximately 180 m from the shoreline with the waves breaking generally on the outer slope of the bar. Consequently the alongshore averaged VLF-intensity normalized with its cross-shore maximum, $U^*_{rms,vlf}$, as a function of cross-shore distance normalized with the surfzone width of 180 m shows maximum VLF-intensity offshore and negligible variations within the surfzone (lower left panel of Figure 3). Hence, the cross-shore distribution of $U_{rms,vlf}$ is unlike the observations that display a clear maximum within the surfzone followed by a strong decay in the offshore direction (compare with Figure 2).

Next, the influence of the rip-channel configuration is examined. A first modification is the introduction of an additional rip-channel with an alongshore separation length of $O(100)$ m, consistent with the RIPEX bathymetry (compare left panel of Figure 1 and
right middle panel of Figure 3). The rip-channel is a mirror image of the first channel located at the same cross-shore location. The presence of the second channel results in interacting VLF motions generated in both channels (middle right panel of Figure 4) causing an onset of the instabilities closer to shore compared to the case with a single rip-channel (compare with left middle panel of Figure 3). As a result, the cross-shore distribution of $U_{rms,vlf}$ shows an increased intensity just outside the surfzone (lower right panel of Figure 3) compared to the case with a single rip channel.

Next, the second rip-channel is given a small offset in the offshore direction emulating the RIPEX experiment, to examine the effect of asymmetry in the rip-channel configuration (left panels of Figure 4). Apparently the asymmetry in the rip-channel configuration makes the system more conducive to the generation of VLF-motions. As a result, the VLF-motions are now pressed a little closer to the surfzone edge than in the case of two symmetric rip channels. Still $U_{rms,vlf}$ within the surfzone is significantly less than outside the surfzone, contrary to the field observations (Figure 2).

As a next step the actual measured bathymetry is used in the model computations to assess the effects of the added bathymetric variability on the VLF-motions. In contrast to the previous cases there is no longer a clear rip-current present that extends well beyond the surfzone (right middle panel of Figure 4) and vorticity is mostly restricted to the surfzone. In this case shear instabilities are generated within the surfzone, however there is no significant offshore decay of the VLF-motions, and consequently the corresponding VLF-intensity within the surfzone is comparable to the offshore VLF-intensity (lower right panel of Figure 4).

In line with the work of Chen at al. [1999] and Haas et al. [2003], the previous results suggest that the underlying bathymetry does play an important role in the generation of VLF-motions reflected in the differences in the cross-shore distribution of the VLF-intensities for different rip channel configurations. However, none of the computed cross-shore distributions match the observed distribution (Figure 2),
suggesting additional mechanisms are present.

**Bi-chromatic Wave Forcing**

For the asymmetric rip-channel configuration, different wave forcing regimes are examined. The monochromatic wave forcing (discussed above) is compared with regular wave groups made up of two incident waves of slightly different frequency and different direction, i.e., bichromatic wave forcing. This results in a spatial variation of the wave energy in both cross-shore and alongshore direction. This spatially and temporally modulated wave field can generate alongshore-periodic slowly migrating rip-currents on an alongshore uniform bathymetry as demonstrated by Fowler and Dalrymple [1990]. Here the effects of intersecting wave trains on the generation of VLF-motions are examined on a rip-channelled beach. Two wave trains with frequencies 0.099 Hz and 0.101 Hz, incident from different directions $\theta_1 = -1.24^\circ$ and $\theta_2 = 40^\circ$ with respective wave amplitudes of 0.49 m and 0.10 m, generate an alongshore modulated wave field with an alongshore spacing of 163 m (upper left panel of Figure 5) and a group period of 500 s that is within the VLF-frequency range. The shear component of the radiation stress made up by the two wave trains is negligible ($\sum S_{xy} = 0$), hence, there is no driving of a mean longshore current. A snapshot of the vorticity field shows strong eddy-circulations within the surfzone (left middle panel of Figure 5). In addition, eddies have separated from the surfzone and are propagating offshore. The cross-shore distribution of $U_{rms, vlf}$ shows a clear maximum within the surfzone at $X^* = 0.75$, decaying in the offshore direction not unlike the observations (compare lower left panel of Figure 5 with upper panel of Figure 2). The VLF intensity outside the surfzone is still over-predicted, suggesting that the offshore eddy formation is too strong. This may be related to the time-scale of the forcing. If the difference frequency of the two intersecting wave trains is small, there is enough time for the rip-current to develop and reach offshore. Once the wave-group forcing changes, this rip-current is then cut-off from
the surfzone, and what remains are eddy like features travelling offshore. In contrast, the time-scale of wave-groups made up of directionally spread incident waves is typically much shorter, i.e. in the infragravity band. As a result the forcing of the rip-current is constantly changing, and the resulting rip-current eddies are expected to remain closer to the surfzone. This is examined next for the case of irregular wave groups.

**Directionally-broad Stochastic Wave Forcing**

A stochastic description of the spatially and temporally varying wave forcing is obtained by considering random wave groups made up of directionally spread incident waves (Reniers et al., 2004a). The wave-action at the offshore boundary for a Jonswap spectrum with a root mean square wave height, $H_{rms}$ of 1.0 m, and a peak period, $T_p$, of 10 s, in combination with a $cos^s(\theta - \bar{\theta})$- directional spreading function with $s = 20$, centered around $\bar{\theta} = 0^\circ$ is obtained with the method outlined by Van Dongeren et al. [2003]. Introducing this formulation into the wave-action balance, (eq.2), results in a strikingly different picture of the spatial variation of the wave energy on the wave-group scale, both in the alongshore and cross-shore directions (upper right panel of Figure 5). The wave energy has the appearance of random blobs incident on the shoreline, where upon breaking, result in radiation stress gradients. The spatial modulation of the wave energy generates both infragravity motions and VLF-motions. The vorticity associated with the VLF-motions, obtained by low-pass filtering the velocities using a cut-off frequency of 0.004 Hz, is indeed mostly confined to the surfzone (middle right panel of Figure 5). The corresponding cross-shore distribution of $U_{rms,vlf}$ shows a maximum within the surfzone, which quickly drops off in the offshore direction. This drop-off occurs sooner compared to the case with bi-chromatic wave groups (compare lower panels of Figure 5).

The fact that the modelled cross-shore distribution of $U_{rms,vlf}$ is similar to the observations suggest that the most important processes are present in the numerical
modelling. A quantitative comparison with the observations is performed in the following.

**Comparison with observations**

Wave boundary conditions for the model are obtained from the offshore directional wave rider buoy, located in 17 m water depth (Reniers et al., 2006), and the tidal elevation is obtained from a NOAA/NOS wave gauge located near the Monterey harbor 2 km south of the experiment site. Hourly estimated frequency-directional spectra are used to generate time series for the spatially and temporally varying wave action using a single summation method (van Dongeren et al., 2003). Using the measured bathymetry, which is periodically extended in the alongshore direction to a total length of 1100 m to mitigate boundary effects, the wave and flow field on the wave-group scale are computed. Time step and grid spacing are the same as for the idealized cases.

The computed time-series of the flow velocities and surface elevation at Puv11 for yearday 130, hours 15-20, show similar signals compared with the measurements in both on the LF and VLF-time scales (compare right panels of Figure 1 with Figure 6). The root mean square wave height at this time is approximately 1 m with a mean wave period of 10 s and normal incidence. Note that the model is run in a stochastic mode, i.e. the wave action boundary signal is constructed from the measured frequency-directional wave spectrum as opposed to utilizing the measured surface elevation time series at the offshore buoy. The computed and measured signals are therefore not expected to match in a deterministic sense, but in a statistical sense.

A comprehensive comparison of the model computations with the measured LF-motions for the duration of the RIPEX experiment expressed in a skill factor (e.g. Gallagher et al., 1998) is given by Reniers et al. [2006], which resulted in a model skill of 0.85 for the incident rms wave height, $H_{rms,hi}$, 0.81 for the rms infragravity wave height, $H_{rms,lo}$, and model skills of 0.76 and 0.65 for the rms low-frequency velocity motions,
$u_{rms,lo}$ and $v_{rms,lo}$. These results demonstrate that wave and roller energy dissipation are well calibrated. However, in addition to the wave forcing, the mean velocity field, responsible for transporting the VLF-motions is important and is assessed next.

The computed mean flow pattern exhibits strong offshore directed flows within the rip-channels reaching velocities up to 0.8 m/s (left panel of Figure 7). The northern rip-channel, located at $Y = 20$ m, has a more or less closed circulation cell, where water carried offshore by the rip-current returns over the relatively shallow shoal. In contrast, the southern rip-channel, located at $Y = 125$ m, carries most of the excess water associated with the mass-flux of the incident sea/swell waves offshore without returning. A comparison with the measured mean velocities shows reasonable correspondence (right panel of Figure 7), suggesting the mean motions are well represented by the model computations at this time.

Given the fact that the offshore extent of shear instabilities is strongly affected by the underlying mean flow field a comparison is made between measured and computed two-hour mean velocity through the surfzone (Figure 8). The mean flow velocities within the surfzone are significantly larger than outside the surfzone (compare results for puv1 with puv4 in Figure 8). In fact, only during the storm conditions on yearday 122 the offshore sensors show mean flow velocities above 0.25 m/s. This is consistent with the limited offshore extent of the VLF-motions observed during RIPEX. The cross-shore deceleration of the mean velocity is well represented by the numerical results, although significant differences at individual sensors between measured and computed mean velocities can exist.

The VLF-motions are examined in the following starting with the cross-shore and alongshore distributions of the daily averaged $U_{rms,vlf}$. The computed cross-shore distribution of $U_{rms,vlf}$ for yearday 130 shows a maximum intensity within the surfzone and subsequent decay in the offshore direction, not unlike the measurements (left upper panel of Figure 9), although the surfzone VLF-intensities are underpredicted.
The corresponding alongshore distribution matches the observations away from the rip-channel, and underpredicts the intensities close to the rip channel. The overall comparison for yearday 130 is satisfactory. Similar results are obtained for yearday 122, during the peak of the storm with an offshore root mean square wave height in the order of 2 m, again with a strong decay of the VLF-intensity in the offshore direction (upper right panel of Figure 9). The computed VLF-intensities in the alongshore direction show less variability than the measurements, but are again of the right order of magnitude (lower right panel of Figure 9).

To obtain a synoptic view of the VLF-motions, the computed mean velocity and mean vorticity field (left panel Figure 7) are subtracted from the calculations. Next the VLF-velocity model computations are low-pass filtered with a frequency cut-off of 0.004 Hz eliminating most of the infragravity contributions within the velocity signal, thus retaining the VLF-velocity response only.

Two snapshots (Figure 10), separated by a 16 minute interval, of the VLF-velocity field and corresponding vorticity show the presence of VLF-motions in both rip-channels with length scales comparable to the rip-channel spacing. The vorticity is concentrated along the 1.5 m depth contour, which corresponds to the location with most intense wave breaking. Closer to the shoreline smaller pockets of vorticity can be observed. Inspection of the temporal evolution (not presented) shows that the VLF-motions in the rip-channel located at $Y = 20$ m are more or less trapped within the circular mean flow circulation (left panel of Figure 7). As a result they do not propagate away from the surfzone, causing strong oscillations of the rip-cell as a whole. In contrast, the VLF-motions within the rip-channel located around $Y = 100$ m propagate with the mean rip-current in the offshore direction and occasionally shoot offshore. The VLF-motions are predominantly initiated at the locations of intense wave breaking, i.e. induced by groups of breaking waves along the 1.5 m depth contour. In the rip-channel located around $Y = 100$ m, oscillations are sometimes initiated close to the shore.
and grow in intensity as they travel offshore, consistent with rip-current instabilities. Interaction between the VLF’s in both channels is apparent, typically resulting in a sequence of alternating cells of positive and negative vorticity (Figure 10). These VLF-circulation cells, or eddies, generally have an offset in the cross-shore direction, resulting in a wave-like pattern for the cross-shore velocity field along the alongshore measurement array, whereas the alongshore velocity field along the array is more or less homogeneous (Figure 10). As a result, the alongshore length-scales of the VLF-motions are quite different for the cross-shore and alongshore velocities, which are examined next.

Applying a 2D-FFT on the computed cross-shore and alongshore velocity time series over the alongshore model domain with a duration of approximately 4 hours, subdivided into 30 minute sections, results in a frequency resolution of 5.5e-4 Hz with 16 degrees of freedom. The resulting frequency wave number, \( f - k_y \), spectra (upper panels of Figure 11) show the presence of infragravity waves, both leaky (in the cross-shore velocity, upper left panel) and trapped (along the edge-wave curves for the alongshore component, upper right panel). In the VLF-frequency band, i.e. below 0.004 Hz, there is significant energy density present outside the gravity restoring region (outside the zero-mode edge wave dispersion curves) associated with VLF-vorticity motions. The corresponding alongshore-scale, \( k_y \), for the cross-shore VLF-motions (upper left panel) matches the rip-channel separation scale of \( \sim 125 \text{m} \) (left panel Figure 1). In contrast the alongshore VLF-velocity energy density is distributed around \( k_y = 0 \text{ m}^{-1} \) (upper right panel). This difference in length scale for the VLF-motions is a result of the way the surfzone eddies interact (Figure 10) as suggested by MacMahan et al. [2004b]. The computed VLF-spectra compare well with the VLF-result obtained with an Iterative Maximum Likelihood Estimator (IMLE) (Pakwa, 1983) utilizing the alongshore array measured velocities (lower panels of Figure 11). This holds for both cross-shore velocities (compare left panels of Figure 11) and alongshore velocities (compare right panels of
The IMLE-spectra for the computed VLF-velocities (not shown) yields similar results as obtained with the 2D-FFT, substantiating the implied assumption of spatial homogeneity of the VLF-flow field, when calculating the spectra.

The effects of temporal changes in incident wave height and tidal elevation are examined by comparing the computed VLF-intensities with the observations for a 10 day period (Figure 12). The VLF-response is clearly related to the incoming wave height with increased intensities of VLF-motions during storm conditions centered around yearday 122 as observed by MacMahan et al. [2004b]. During calmer wave conditions, the VLF-response is weaker. This behavior is also present in the computations. The observed spatial variability in response, compare puv9 with puv2, is also present in the computed results. MacMahan et al. [2004b] observed a VLF-response on the tidal scale (e.g. puv9 yearday 123 to 126, Figure 12), where \( U_{rms,clf} \) is modulated similar to the mean currents with increased velocity at low tide. Finally, the measured offshore decay of the VLF-intensities is well represented by the computations as reflected by puv’s 5 (within the surfzone), 3 (at the surfzone edge) and 4 (outside the surfzone) throughout the ten day period. The average model skill for the sensors within the surfzone is 0.7 (Table 1).

### Discussion

The present numerical results for monochromatic waves (e.g. left panels of Figure 3) contrast with earlier laboratory observations of surfzone VLF’s (Haller and Dalrymple, 2001), with strong VLF motions located within the surfzone followed by a rapid decay in the offshore direction. The onset and the growth rate of shear instabilities in a rip-current circulation are strongly dependent on the combination of the forcing by the velocity shear and damping mechanisms such as bottom friction and turbulent eddy viscosity (Haller and Dalrymple, 2001). The velocity shear within the rip-channel for the laboratory conditions is an order of magnitude larger than the corresponding velocity
shear during RIPEX (MacMahan et al., 2006). Hence, the shear instability forcing is much stronger in the laboratory than in the RIPEX field case. This results in an onset of the instabilities within the surfzone, and rapid growth of the shear instabilities while propagating with the rip-current. Compared to the field case, the steeper beach slope for the laboratory conditions results in a stronger deceleration of the rip current flow, thus limiting the offshore extent of the VLF motions that are travelling with the outgoing rip current. In addition, the mixing induced by the finite amplitude shear instabilities disperses the outgoing rip-current flow, thereby restricting the offshore extent of the VLF motions further. This surfzone dominance and limited offshore extent of the VLF motions was confirmed by numerical simulations (Haas et al., 2003, Chen et al., 1999). In contrast, the smaller velocity shear for the RIPEX-configurations presented here delays the onset of the instabilities to the outer surfzone, where damping associated with turbulent eddy viscosity and friction is relatively small compared to the inner surfzone where wave breaking induced turbulence (viz. eq. 13) and bottom friction are important. The corresponding smaller growth rate results in persisting rip-currents which take the shear instabilities offshore, well away from the surfzone.

Conclusions

Field observations of very low frequency motions (VLF’s) have been compared with a non-linear shallow water model operating on the wave-group time-scale. The model comparisons are made for the three different wave forcing regimes of monochromatic waves, bi-chromatic waves, and broadband frequency and directional waves, all having the same $H_{rms}$ of 1 m, mean period of 10 seconds with $0^\circ$ mean wave angle. The model constituents of turbulent eddy viscosity, bottom friction in combination with wave-current interaction were determined quantitatively based on comparisons of simulated and observed cross-shore and alongshore velocity intensity of the VLF’s and mean flow strength. This resulted in a single parameter set that has been used for all
model computations on which the conclusions presented below are based.

Model computations show that the most important contribution to explain the observed VLF motions during RIPEX is the temporal variability of the wave forcing on the group scale. For the case of monochromatic waves, only long-crested waves are incident resulting in strong wave-current interactions; VLF motions are limited to instabilities of the rip current itself and result in the VLF energy displaced too far offshore compared with measurements. Bi-chromatic incident wave energy is periodic alongshore. Strong eddy circulation occurs within the surf zone as the result of wave breaking of the wave groups. The VLF-velocities intensities are confined closer to the shoreline compared with the mono-chromatic case.

The forcing by directionally-broad random waves can explain the high $U_{\text{rms, vlf}}$ in the surf zone and the rapid decay of the intensity in the offshore direction just outside the surfzone during RIPEX. The waves are derived from the measured directional spectra offshore, which drive the waves on the wave-group scale. The incident wave field appears as ”random blobs” of energy with alongshore length scales O(200m), forcing VLF-motions on the scale of the rip channel spacing. The results are consistent with surf zone eddies as hypothesized by MacMahan et al., 2004b. Model predicted VLF-velocity intensities forced stochastically by wave energy obtained from the measured offshore spectra are compared with measured values. The VLF-velocity intensities are predicted with an average skill of 0.7.

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References


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This manuscript was prepared with AGU’s \LaTeX \text{ macros v4, with the extension package 'AGU++' by P. W. Daly, version 1.6b from 1999/08/19.}
Figure Captions

Figure 1. Left panel: Survey for yearday 131 with instrument locations denoted by squares (collocated pressure and current meters denoted by puv) and dots (pressure meters only denoted by p). Right panel: Example of measured time series (thin solid lines) of low-pass filtered ($f < 0.04$ Hz) cross-shore velocity (upper plate), alongshore velocity (middle plate) and surface elevation (lower plate), showing the presence of infragravity motions superimposed on VLF-motions (thick dashed white line) for PUV11.
Figure 2. Top panel: Example of measured cross-shore distribution of daily averaged VLF-intensity (denoted by the dots) on yearday 130. Position of alongshore array indicated by the arrow. Cross-shore bottom profile is given as a reference by the thick dashed line. Bottom panel: Similar for the alongshore distribution. Position of shoal array denoted by arrow. Alongshore bottom profile is given as a reference by the thick dashed line.
Figure 3. Left panels: Snapshot of wave energy (upper panel), corresponding vorticity (middle panel) and normalized alongshore-averaged VLF-intensity, $U_{rms,vlf}^*$, as function of normalized cross-shore distance, $X^*$, for monochromatic waves with $H = 1$ m and $T = 10$ s, normally incident on a coast with a single rip-channel, with wave-current interaction. Right panels: Similar for the case of two symmetric rip-channels.
Figure 4. Left panels: Snapshot of wave energy (upper panel), corresponding vorticity (middle panel) and normalized alongshore-averaged VLF-intensity, $U_{rms,vlf}^*$, as function of the normalized cross-shore distance, $X^*$, for regular waves with $H = 1$ m and $T = 10$ s, normally incident on a coast with two asymmetric rip channels (with an offset in the cross-shore direction). Right panels: Similar for the case of the actual bathymetry.
Figure 5. Left panels: Snapshot of wave energy (upper panel), corresponding vorticity (middle panel) and normalized alongshore-averaged VLF-intensity, $U_{\text{rms, vlf}}^*$, as function of normalized cross-shore distance, $X^*$, for bi-chromatic wave groups made up of two intersecting waves with a mean frequency of 0.1 Hz and a frequency difference of 0.002 Hz and zero mean wave direction. Right panels: Similar for wave groups made up of a directionally spread Jonswap spectrum with $H_{\text{rms}} = 1.0$ m, $T_p = 10$ s and zero mean direction.
Figure 6. Example of computed time series of low-pass filtered ($f < 0.04$ Hz) cross-shore velocity (upper plate), alongshore velocity (middle plate) and surface elevation (lower plate), showing the presence of infragravity motions (thin line) superimposed on VLF-motions (thick dashed white line) for yearday 130, hours 15-20.
Figure 7. Left panel: Computed 2 hour averaged flow velocity (see arrow for scaling) and corresponding vorticity in $s^{-1}$ for yearday 130 hour 16. Depth contours in meters given by the solid white lines and instrument locations by the white squares. Right panel: Comparison of measured mean flow velocities (white arrows) with computed mean velocities (black arrows) for yearday 130 hour 16.
Figure 8. Computed (solid line) and measured (dots) 2-hour mean velocity magnitudes for a 10 day period at various sensors traversing the surfzone (see Figure 1 for locations). Tidal elevation is given in the lowest panel as a reference.
Figure 9. Left top panel: Comparison of measured (dots) and computed daily averaged cross-shore VLF-intensity for yearday 130 (squares). Bottom left panel: Similar, but for the alongshore distribution. Right top panel: Comparison of measured (dots) and computed daily averaged cross-shore VLF-intensity for yearday 122 (squares). Bottom right panel: Similar, but for the alongshore distribution.
Figure 10. Snapshots of computed VLF-velocity response at two separate intervals separated by 16 minutes showing the VLF-velocity field (see arrow for scaling) and corresponding vorticity field $q$ in s$^{-1}$ for yearday 130 hour 16. Depth contours in meters given by the solid white lines and instrument locations by the white squares.
Figure 11. Upper panels: $f - k_y$-spectra computed with 2DFFT for the computed cross-shore velocity (left panel) and alongshore velocity for yearday 130 hours 16-20 at the alongshore array position ($X = 88$ m). Lower panels: $f - k_y$-spectra computed using IMLE for the measured cross-shore velocity (left panel) and alongshore velocity for yearday 130 hours 16-20 at the alongshore array. The dashed and dash-dotted lines indicate the zero and first mode edge wave dispersion curves and the area inbetween the solid lines represent the leaky wave regime. The horizontal dotted line at $f = 0.004$ Hz indicates the VLF-frequency cut-off.
Figure 12. Comparison of measured VLF-intensity (dots) with computed results for a ten day period at various sensor locations. Tidal elevation is given in the lowest panel as a reference.
### Table 1
Skill factors for predicted VLF-intensity at various sensor locations (see left panel of Figure 1) within the surfzone during yeardays 121-130.

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<th>sensor</th>
<th>skill (WCI)</th>
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<td>puv9</td>
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