P-Vector Method for Determining Arctic Ocean Circulation from the Joint US-Russian Hydrographic Data

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References


Motivation

• Improving the weakness of diagnostic initialization

• Numerical model is usually integrated from \((T, S)\) field (climatology or …)
  \[ u = v = w = 0 \]

Diagnostic Initialization
Basic Equations for OGCM

\[
\frac{\partial V}{\partial t} = -V \cdot \nabla V - w \frac{\partial V}{\partial z} - k \times fV - \frac{1}{\rho} \nabla p + \frac{\partial}{\partial z} \left( K_M \frac{\partial V}{\partial z} \right) + H_v
\]

\[
\frac{\partial T}{\partial t} = -V \cdot \nabla T - w \frac{\partial T}{\partial z} + \frac{\partial}{\partial z} \left( K_H \frac{\partial T}{\partial z} \right) + H_T,
\]

\[
\frac{\partial S}{\partial t} = -V \cdot \nabla S - w \frac{\partial S}{\partial z} + \frac{\partial}{\partial z} \left( K_H \frac{\partial S}{\partial z} \right) + H_S,
\]
Diagnostic Initialization

\[
\frac{\partial T}{\partial t} = -\mathbf{V} \cdot \nabla T - w \frac{\partial T}{\partial z} + \frac{\partial}{\partial z} (K_H \frac{\partial T}{\partial z}) + H_T + F_T,
\]

- Keep (T, S) constant

\[
\frac{\partial S}{\partial t} = -\mathbf{V} \cdot \nabla S - w \frac{\partial S}{\partial z} + \frac{\partial}{\partial z} (K_H \frac{\partial S}{\partial z}) + H_S + F_S,
\]

- Generate (u, v, w) fields

\[
\frac{\partial T}{\partial t} = 0, \quad \frac{\partial S}{\partial t} = 0
\]
Extremely Strong Thermohaline Source/Sink Terms (Non-Physical)

\[ F_T = \mathbf{V} \cdot \nabla T + w \frac{\partial T}{\partial z} - \frac{\partial}{\partial z} \left( K_H \frac{\partial T}{\partial z} \right) - H_T, \]

\[ F_S = \mathbf{V} \cdot \nabla S + w \frac{\partial S}{\partial z} - \frac{\partial}{\partial z} \left( K_H \frac{\partial S}{\partial z} \right) - H_S. \]

Geostrophic Initialization

- Absolute geostrophic velocity computed from hydrographic data

\[ (T, S) \rightarrow \text{Density} \rightarrow \text{Velocity} \]
First Thought - Thermal Wind Relation

\[ u = u_0 + \frac{g}{f\rho_0} \int_{z_0}^{z} \frac{\partial \rho}{\partial y} \, dz' \]

\[ v = v_0 - \frac{g}{f\rho_0} \int_{z_0}^{z} \frac{\partial \rho}{\partial x} \, dz' \]

How to determine \((u_0, v_0)\)?
Conservation of Mass and Potential Vorticity

\[ \nabla \cdot \nabla \rho = 0 \]

\[ \vec{V} \cdot \nabla q = 0, \quad q \equiv f \frac{\partial \rho}{\partial z} \]
Relationship Among Three Vectors

\[ \vec{V} \perp \nabla \rho \quad \vec{V} \perp \nabla q \]

\[ \vec{V} \sim \nabla q \times \nabla \rho \]
P-Vector

\[ \vec{P} = \frac{\nabla q \times \nabla \rho}{|\nabla q \times \nabla \rho|} \]

\[ \vec{P} \parallel \vec{V} \]

\[ \vec{V} = r(\lambda, \phi, z)\vec{P} \]
Two-Step Inverse Method

(1) Density determines the P-vector.

(2) Thermal wind relation determines $\gamma$. 
P-Vector
Intersection of density and potential vorticity surfaces
Thermal Wind Relation

Determines \( \gamma \)

\[
\begin{align*}
   r^{(k)} P_x^{(k)} - r^{(m)} P_x^{(m)} &= \Delta u_{km} \\
   r^{(k)} P_y^{(k)} - r^{(m)} P_y^{(m)} &= \Delta v_{km}
\end{align*}
\]

\[
\Delta u_{km} \equiv \frac{g}{f \rho_0} \int_{z_m}^{z_k} \frac{\partial \rho}{\partial y} \, dz'
\]

\[
\Delta v_{km} \equiv -\frac{g}{f \rho_0} \int_{z_m}^{z_k} \frac{\partial \rho}{\partial x} \, dz'
\]
Solution of $\gamma$

$$r^{(k)} = \frac{\begin{vmatrix} \Delta u_{km} & P_x^{(m)} \\ \Delta v_{km} & P_y^{(m)} \end{vmatrix}}{\sin(\alpha_{km})}$$

$$\begin{vmatrix} P_x^{(k)} & P_x^{(m)} \\ P_y^{(k)} & P_y^{(m)} \end{vmatrix} = \sin(\alpha_{km})$$

$\alpha_{km} \neq 0$
US-Russian Arctic Hydrographic Data

http://nsidc.org/
Temperature \((z = 0)\)
Temperature ( \( z = -1000 \) m)
Salinity \((z = 0)\)
Salinity ($z = -1000 \text{ m}$)
Mean Circulations ($z = -32 \text{ m}$)

P-vector

7 years mean of POPS model results (Zhang, Maslowski, Semtner, 1999, JGR)
Mean Circulations \((z = -32 \text{ m})\)

P-vector

7 years mean of POPS model results (Zhang, Maslowski, Semtner, 1999, JGR)
Conclusions

• (1) P-Vector method is effective to calculate the absolute velocity from hydrographic data.

• (2) The initial velocity filed can be calculated using the P-vector method (Geostrophic Initialization).