Nonlinear Model
Predictability

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References


Physical Reality

- $Y$

- Physical Law: $\frac{dY}{dt} = h(y, t)$

- Initial Condition: $Y(t_0) = Y_0$
Atmospheric Models

- $X$ is the prediction of $Y$

- $\frac{dX}{dt} = f(X, t) + q(t) X$

- Initial Condition: $X(t_0) = X_0$

- Stochastic Forcing:
  - $\langle q(t) \rangle = 0$
  - $\langle q(t)q(t') \rangle = q^2\delta(t-t')$
Model Error

\[ Z = X - Y \]

Initial: \[ Z_0 = X_0 - Y_0 \]
One Overlooked Parameter

- Tolerance Level $\varepsilon$
- Maximum accepted error
Valid Predict Period (VPP)

- VPP is defined as the time period when the prediction error first exceeds a pre-determined criterion (i.e., the tolerance level $\varepsilon$).
First-Passage Time
Conditional Probability Density Function

- Initial Error: \( Z_0 \)

- \((t - t_0) \uparrow\downarrow \) Random Variable

- Conditional PDF of \((t - t_0)\) with given \( Z_0 \)

  - \( P[(t - t_0) | Z_0] \)
Two Approaches to Obtain PDF of VPP

- Analytical (Backward Fokker-Planck Equation)
- Practical
Backward Fokker-Planck Equation

\[
\frac{\partial P}{\partial t} - \left[ f(z_0, t) \right] \frac{\partial P}{\partial z_0} - \frac{1}{2} q^2 z_0^2 \frac{\partial^2 P}{\partial z_0 \partial z_0} = 0
\]
\[ \tau_1(z_0) = \int_{t_0}^{\infty} P(t_0, z_0, t-t_0)(t-t_0) \, dt \]

\[ \tau_2(z_0) = \int_{t_0}^{\infty} P(t_0, z_0, t-t_0)(t-t_0)^2 \, dt \]
Extremely Long Predictability

- Non-Gaussian Distribution with Long Tail Towards Large FPT Domain.
Gulf of Mexico Forecast System

- University of Colorado Version of POM
- 1/12° Resolution
- Real-Time SSH Data (TOPEX, ESA ERS-1/2) Assimilated
- Real Time SST Data (MCSST, NOAA AVHRR) Assimilated
- Six Months Four-Times Daily Data From July 9, 1998 for Verification
Model Generated Velocity Vectors at 50 m on 00:00 July 9, 1998
(Observational) Drifter Data at 50 m on 00:00 July 9, 1998
Statistical Characteristics of VPP for zero initial error and 22 km tolerance level (Non-Gaussian)
Probability Density Function of VPP calculated with different tolerance levels

Non-Gaussian distribution with long tail toward large values of VPP (Long-term Predictability)
Error Mean and Variance

Error Mean

\[ L_1 = \langle z \rangle \]

Error Variance

\[ L_2 = \left\langle \left( z - \langle z \rangle \right)^t (z - \langle z \rangle) \right\rangle \]
Exponential Error Growth

\[ L_1 \propto e^{\sigma t}, \quad L_2 \propto e^{\omega t}, \]

Classical Linear Theory

No Long-Term Predictability
Power Law

\[ L_1 \propto t^\alpha, \quad L_2 \propto t^\beta, \]

\[ P(t_0, z_0, \varepsilon, t-t_0) \sim t^{-\gamma} \quad \text{for large} \ t. \]

Long-Term Predictability May Occur
Scaling behavior of the mean error ($L_1$) growth for initial error levels:

(a) 0 km

(b) 2.2 km

(c) 22 km
Scaling behavior of the Error variance ($L_2$) growth for initial error levels:

(a) 0

(b) 2.2 km

(c) 22 km
Conclusions

- (1) Nonlinear model predictability can be effectively represented by FPT.

- (2) Backward Fokker-Planck equation is the theoretical framework for FPT.

- (3) Nonlinear stochastic-dynamic modeling