A Finite Volume Coastal Ocean Model

Peter C Chu and Chenwu Fan
Naval Postgraduate School
Monterey, California, USA
Four Types of Numerical Models

- Spectral Model (not suitable for oceans due to irregular lateral boundaries)
- Finite Difference (z-coordinate, sigma-coordinate, …)
- Finite Element
- Finite Volume
Finite Volume
Finite Volume Model

- Transform of PDE to Integral Equations
- Solving the Integral Equation for the Finite Volume
- Flux Conservation
Dynamic and Thermodynamic Equations

• Continuity

\[ \nabla \cdot (\rho \mathbf{V}) = 0 \]

• Momentum

\[ \frac{\partial (\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) = -\nabla \cdot \tau + \nabla \cdot (\kappa \phi \nabla \phi) + \mathbf{F} \]

• Thermodynamic

\[ \frac{\partial \phi}{\partial t} + \nabla \cdot (\mathbf{V} \phi) = \nabla \cdot (\kappa_\phi \nabla \phi) + F_\phi \]
Integral Equations for Finite Volume

- **Continuity**
  \[ \int_{\Omega} \nabla \cdot (\rho V) \, d\Omega = \oint_{\Gamma} \rho V \cdot n \, d\Gamma = 0 \]

- **Momentum**
  \[ \int_{\Omega} \frac{\partial (\rho V)}{\partial t} \, d\Omega + \oint_{\Gamma} \rho V V \cdot n \, d\Gamma = - \oint_{\Gamma} p d\Gamma + \oint_{\Gamma} \mu \nabla V \cdot n d\Gamma + \int_{\Omega} F d\Omega \]

- **Thermodynamic**
  \[ \int_{\Omega} \frac{\partial \phi}{\partial t} \, d\Omega + \oint_{\Gamma} \phi V \cdot n \, d\Gamma = \oint_{\Gamma} \kappa \phi \nabla \phi \cdot n \, d\Gamma + \int_{\Omega} F_\phi \, d\Omega \]
Time Integration of Phi-Equation

\[ \int_\Omega \phi(t_2) d\Omega - \int_\Omega \phi(t_1) d\Omega = -\Delta t \int_\Gamma \phi(t^*) V \cdot n d\Gamma \]

\[ + \Delta t \int_\Gamma \kappa_\phi \nabla \phi(t^*) \cdot n d\Gamma + \Delta t \int_\Omega F_\phi(t^*) d\Omega \]
Comparison Between Finite Difference (z- and sigma-coordinates) and Finite Volume Schemes
Seamount Test Case
Initial Conditions

- $V = 0$
- $S = 35$ ppt

$$T(z) = 5 + 15 \exp\left(\frac{z}{H_T}\right) \quad \text{(unit: } ^\circ\text{C})$$

- $H_T = 1000$ m
Known Solution

- \( V = 0 \)
- Horizontal Pressure Gradient = 0
Evaluation

• Princeton Ocean Model
• Seamount Test Case

• Horizontal Pressure Gradient (Finite Difference and Finite Volume)
Numerics and Parameterization

• Barotropic Time Step: 6 s
• Baroclinic Time Step: 180 s
• Delta x = Delta y = 8 km
• Vertical Eddy Viscosity: Mellor-Yamada Scheme
• Horizontal Diffusion: Samagrinsky Scheme with the coefficient of 0.2
Error Volume Transport
Streamfunction

FDE (day 5)
FVM (day 5)
FDE (day 15)
FVM (day 15)
FDE (day 10)
FVM (day 10)
FDE (day 20)
FVM (day 20)

contour from -10 to 27.5 by 2.5 \(10^{-3}\) Sv
contour from -8.75 to 13.75 by 1.25 \(10^{-3}\) Sv
contour from -40 to 72 by 6 \(10^{-3}\) Sv
contour from -24 to 39 by 3 \(10^{-3}\) Sv
contour from -50 to 54 by 4.5 \(10^{-3}\) Sv
contour from -18 to 27 by 2.25 \(10^{-3}\) Sv
contour from -50 to 54 by 7 \(10^{-3}\) Sv
contour from -20 to 40.5 by 3.5 \(10^{-3}\) Sv
Temporally Varying Horizontal Gradient Error

- The error reduction by a factor of 14 using the finite volume scheme.

![Graphs showing maximum and mean pressure gradient error over days for FDE and FVM methods.](image)
Temporally Varying Error Velocity

- The error velocity reduction by a factor of 4 using the finite volume scheme.
Conclusions

• Use of the finite volume model has the following benefit:
  (1) Computation is as simple as the finite difference scheme.
  (2) Conservation on any finite volume.
  (3) Easy to incorporate high-order schemes
  (4) Upwind scheme