UTM and UPS

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I. Introduction

The Universal Transverse Mercator (UTM) projection coordinates occur on most topographic maps. This is the Northing and Easting coordinates discussed below. In addition the projection is used on many charts and maps. It forms the basis for the Military Grid Reference System (MGRS) for US DoD maps in non-polar regions. North of 84 N and south of 80 S the Universal Stereographic Projection (UPS) is used for the MGRS.

Note that as with all projections, the map is not specified until the ellipsoid and datum are specified. You have to check the map legend.

II. Universal Transverse Mercator (UTM)

A. Basic Geometry of a UTM Zone

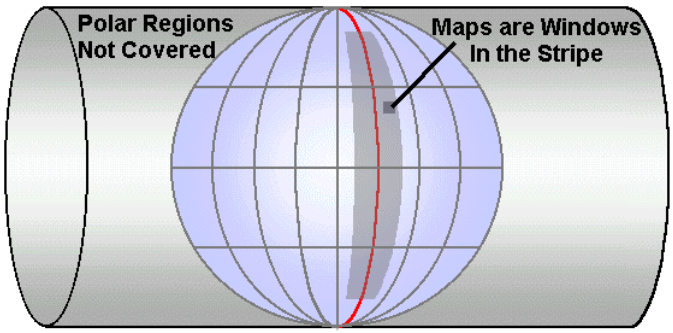
This is a Mercator based on a cylinder that is turned on its side so that the tangent line would be a meridian of longitude. In fact the cylinder is slightly depressed into the earth. The amount is defined by specifying the scale value on the central meridian is 0.9996 . Because it is a Mercator projection, it is conformal. (Rotating the cylinder just rotates Tissot's Indicatrix, which is a circle and remains a circle if you rotate it.) The meridians are now curved lines. The parallels are straight lines. The meridians bow in. This means that azimuths will be in error for points not on the central meridian.

The scale error will be zero on the meridians that cut the earth. These are standard lines. The scale error is, of course 0.0004 (1 - 0.9996) along the central meridian. It grows as you go east or west of the standard lines. The maximum scale error is controlled by only using the projection +/- 3 degrees of longitude of the central meridian.

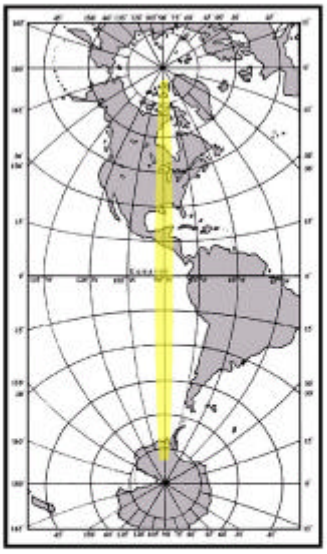
This is a zoned map. There are 60 separate projections, one for each 6 degree band of longitude beginning at 180E. This controls the error. The green line in the distortion graph above is in fact the maximum error for the UTM projection. The first zone covers 180E to 186 E, the second 186 E to 172 E and so forth. Technically the UTM only is defined from 84 N to 80 S.

This is a windowed projection. What is seen on a map, is a part of the projection for the whole zone. Adjacent maps within one zone will line up on the edges. The error is controlled by depressing the cylinder into the earth and by limiting the width of the zone. No point is more than 3 degrees from the **central meridian**.

You can plot the entire earth as a Transverse Mercator projection. This is done on the following figure with the central meridian through the US. Notice that the zone does not extend to the poles. It cuts off at 80 degrees south, but goes to 84 north for the convenience of map makers.

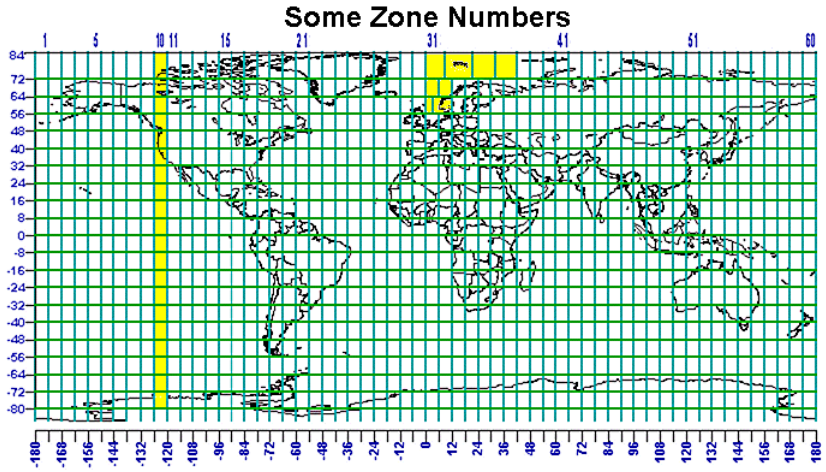


Universal Tranverse Mercator
Cylinder Turned on Side
Used in 6° Wide Stripe or Zone



Transverse Mercator
At 90 W

UTM Zone Width In Yellow
(UTM Zone Offset - 90 Is Edge)



UTM Zones

UTM Longitude Zones with Zone 10 Highlited
Note Irregular Zones in Scandinavia

The first zone, numbered 1, is between longitudes of 180 E and 186 E, or 180 W and 177 W. This means that the area to the east of the prime meridian is zone 31. The official zone map is above. Zone 10 has been colored. In addition the few irregular zones in Scandinavia are also colored. (These

are often incorrectly computed by computer programs.) The horizontal lines every 8 degrees are used in the Military Grid Reference System (MGRS) discussed below.

Zone	West Longitude	Zone	West Longitude	Zone	West Longitude	Zone	West Longitude
1	180 W	16	90 W	31	0 E	46	90 E
2	174 W	17	84 W	32	6 E	47	96 E
3	168 W	18	78 W	33	12 E	48	102 E
4	162 W	19	72 W	34	18 E	49	108 E
5	156 W	20	66 W	35	24 E	50	114 E
6	150 W	21	60 W	36	30 E	51	120 E
7	144 W	22	54 W	37	36 E	52	126 E
8	138 W	23	48 W	38	42 E	53	132 E
9	132 W	24	42 W	39	48 E	54	138 E
10	126 W	25	36 W	40	54 E	55	144 E
11	120 W	26	30 W	41	60 E	56	150 E
12	114 W	27	24 W	42	66 E	57	156 E
13	108 W	28	18 W	43	72 E	58	162 E
14	102 W	29	12 W	44	78 E	59	168 E
15	96 W	30	6 W	45	84 E	60	174 E

B. Scale Value

The construction of a Mercator, and hence also a transverse Mercator can be done geometrically. It was done that way originally. Today we us formulas for the map x and y in terms of the latitude, ϕ, longitude λ and the radius of the globe R. (R is the nominal scale factor times the radius of the earth.) As with the Mercator, the projection is conformal and hence h = k everywhere. Normally only the formula for k are therefore given. The formulas for a sphere are tractable. For the ellipsoid they become quite complex. They are usually written as an expansion in the eccentricity of the earth. Here the formulas for the sphere will be given.

The formula for the scale factor is,

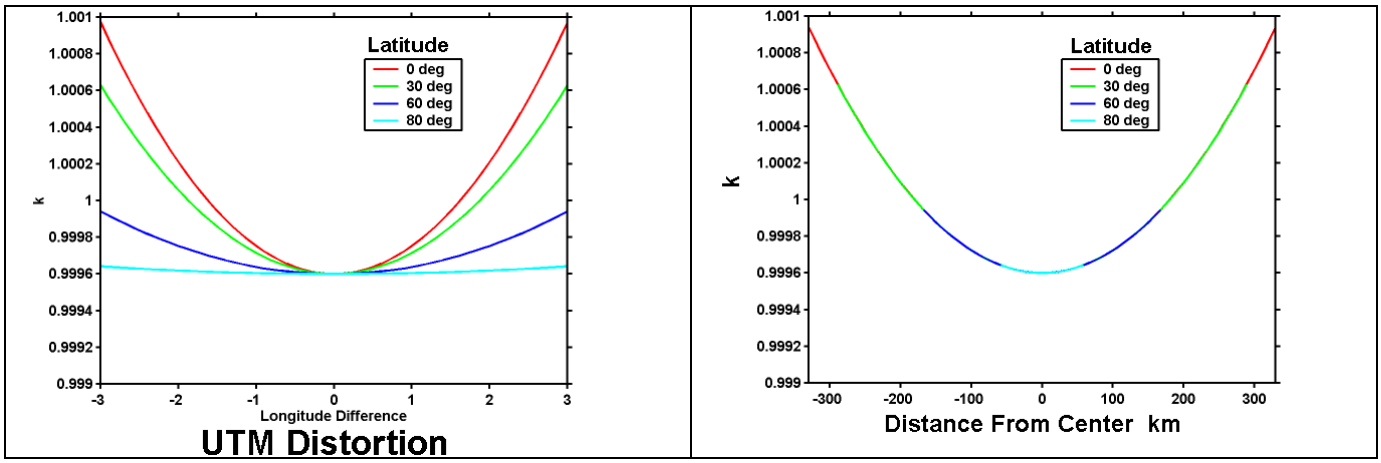
$$k = \frac{k_o}{\sqrt{1-B^2}},$$

where

$$B = \cos\phi \sin(\lambda - \lambda_0).$$

The central meridian is at longitude λ_0 and where the scale factor is k_0. Of course for the UTM k_0 is the value 0.9996 . The maximum value of the longitude difference within a zone is 3 degrees. The sine factor in B is therefore small, having a maximum absolute value of 0.052 . B is always small. Note also that B has a latitude dependence.

The scale value is the most important of these values to understand. It is shown below for four latitudes. On the left is the independent values is the longitude difference. However the distance of a degree of longitude is a function of latitude as the meridians converge at the poles. On the right the same data is plotted as a function of distance from the center. The whole zone width is plotted. The curves overlay in terms of distance. At the center the value is 0.9996 as set by the standard. This means that distances will be 0.40 m short for every kilometer there. At the edge at the equator, about 330 km from the center, the distances will be 0.98 m / km long.



C. Eastings and Northings

The formula for the x and y coordinates are usually written in terms of B.

$$x = Rk_0 \frac{1}{2} \ln\left[\frac{1+B}{1-B}\right]$$

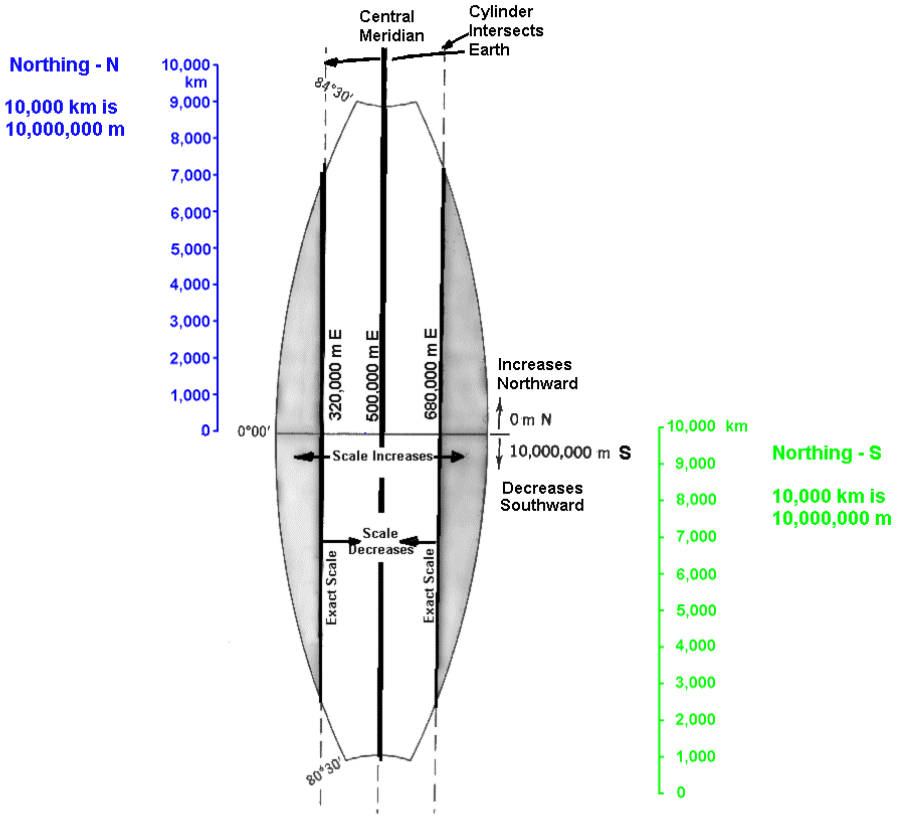
$$y = Rk_0 \arctan\left[\frac{\tan\phi}{\cos(\lambda-\lambda_0)}\right].$$

Here x is the horizontal map coordinate, the one in the east-west direction. If the real radius of the earth is used, then the value is a distance called **Easting**, usually expressed in meters. Easting is zero on the central meridian using the above equation for x. Easting would be both plus and minus. At the time UTM was first used, negative numbers were not well know. To avoid them, an offset was added. The value 500,000 was chosen. This keeps the left or western edge of all points in a zone positive. (The minimum distance on the plot of distortions above is -330 km .)

The values in the north-south direction are also handled to avoid negative numbers. These are called **Northings**. For points north of the equator there is no need for an offset. However south of the equator an offset is needed. The value of ten million meters (10,000,000 m) is added to northings for points south of the equator.

A diagram is shown below of a UTM zone and the Northings. Notice that there will be the same value used in both the north and south hemisphere. There are multiple conventions on how to handle this. On maps you usually attach a "N" or "S" to the values to designate hemisphere. On computers the zone number is set to the negative of its correct value south of the equator for several implementations. You just have to read the documentation with computer programs.

Notice that at high latitudes the cylinder does not break the surface at the zone boundary. This means that the scale error will always be negative at those latitudes. The standard lines do not enter the zone there. This is one of the reasons that the UTM system is not used at high latitudes. There a stereographic projection (UPS) is used. Both UTM and UPS were chosen in part because they are conformal projections.



UTM Zone

Eastings, Northings And Cylinder Position

Eastings of Standard Lines Approximate for Ellipsoid Earth

A table of the UTM coordinates at a few points throughout the zone follow. These were computed on the WGS84 ellipsoid and datum.

Latitude	Zone Location	Northing m	Easting m	Error m/km
+84	Center	9328094	500000	-0.40
+80	Center	8881586	500000	-0.40
+45	Center	4982950	500000	-0.40
0	Center	0.0	500000	-0.40
-45	Center	5017050	500000	-0.40
-80	Center	1118414	500000	-0.40

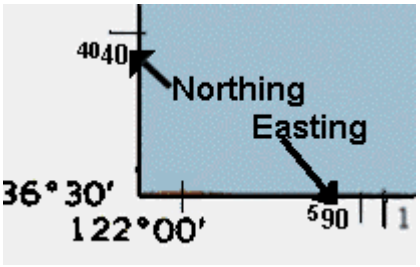
Notice that the Easting at the center is always 500,000 and the scale error 0.4 m / km there. Notice also that the Northings are always increasing as you go north, but jump back 10,000,000 m at the equator. By convention the 0 value is normally used for the equator.

The values of the UTM Northings and Eastings at the edges of the zone are shown in the table below. Notice that the scale error is now larger. It is positive on the zone boundaries for most of the zone. Also notice that the Northings are the same on the edge, but slightly different from the center value in the table above. This is due to the projection. Keeping it conformal has some consequences, even close to the central meridian.

Latitude	Zone Location	Northing m	Easting m	Error m/km
+84	West Edge	9329005	465005	-0.39
	East "	9329005	534995	-0.39
+80	West Edge	8883085	441878	-0.36
	East "	8883085	558132	-0.36
+45	West Edge	4987329	263554	0.29
	East "	4987329	736446	0.29
0	West Edge	0.0	166021	0.98
	East "	0.0	833979	0.98
-45	West Edge	5012670	263554	0.29
	East "	5012670	736446	0.29
-80	West Edge	1116915	441878	-0.36
	East "	1116915	558132	-0.36

D. UTM Grids on Maps

On many topographic maps and some charts a UTM grid of eastings and northings is overlaid. On USGS topographic maps only edge tic marks are present. This is done despite the fact that the projection of the map is not UTM.



The above image of the map that covers Monterey CA shows the UTM tics. These are indicated in a large/small print format. The large number is multiplied by 100,000 m and the small by 10,000 m. Therefore the easting value of the tic is 590,000 m E. This is just bigger than the 500,000 value of the central meridian. The northing value shown is 4,040,000 m N. Thus this point is just over 4 million meters (4000 km) north of the equator along an arc of the ellipsoid.

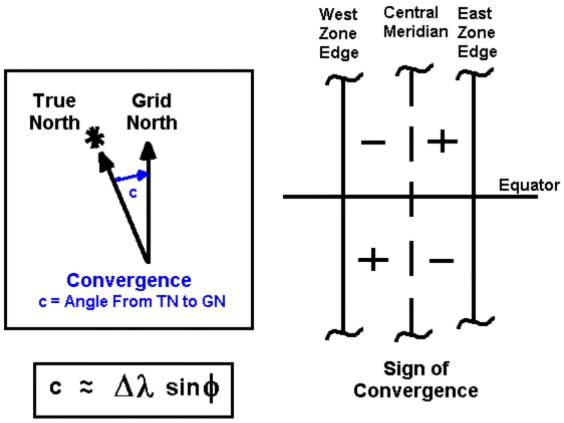
E. Azimuth Error – Convergence

It is obvious from the above diagram showing the zone boundaries and the cylinder edges that the Northing lines do not line up with the true north, except on the equator. This leads to an azimuth error if the differences of Eastings and Northings are used in an inverse tangent function. This difference is called **convergence.**

The convergence is defined as the angle from the **true north** to the **grid north**. Here grid north is the vertical lines or the northing axis. It is approximately given by the equation

$$c \approx \Delta\lambda \sin\phi$$

where the longitude difference is taken in east longitude. It will be positive to the east of the central meridian and negative to the west of the central meridian. This is in line with the bowing in of the true north lines on a UTM map. This equation is approximate even for a spherical earth model. There are corrections for the spherical formula. In either case, the above is in error by 0.003 times the approximate value at maximum. The largest azimuth errors are close to 3 degrees in the polar regions. The grid convergence is usually given in the legend of a navigation chart or topographic map.



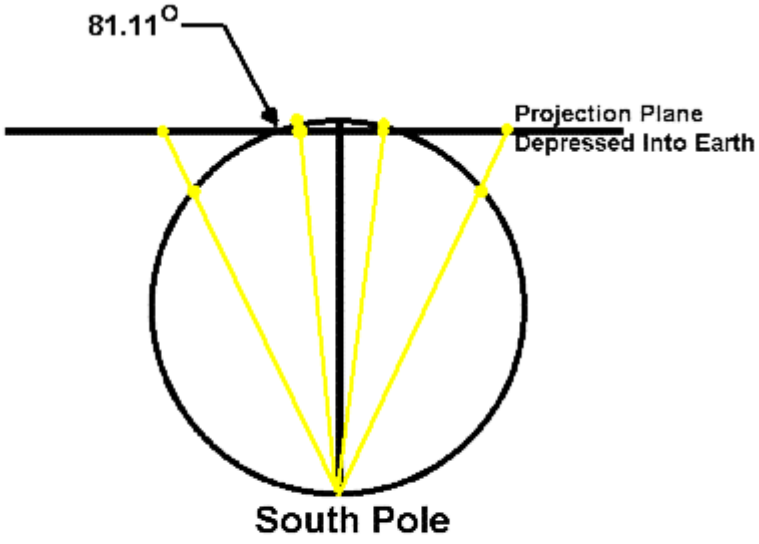
$$c \approx \Delta\lambda \sin\phi$$

UTM Convergence Azimuth Error

III. Universal Stereographic Polar

In the polar regions the UTM system suffers greatly from convergence. Therefore the standard map used there is a stereographic projection. This is chosen because it has reasonable distortion and it is conformal. It is important that a conformal projection be chosen for any projection used for navigation.

The Universal Polar Stereographic projection is defined for latitudes north of 84 N and south of 80 S. The asymmetry is due to boundaries of important land masses. The plane of projection is depressed into the earth. The scale factor at the center is 0.994 . This means the place cuts the earth at about 81.11 degrees.

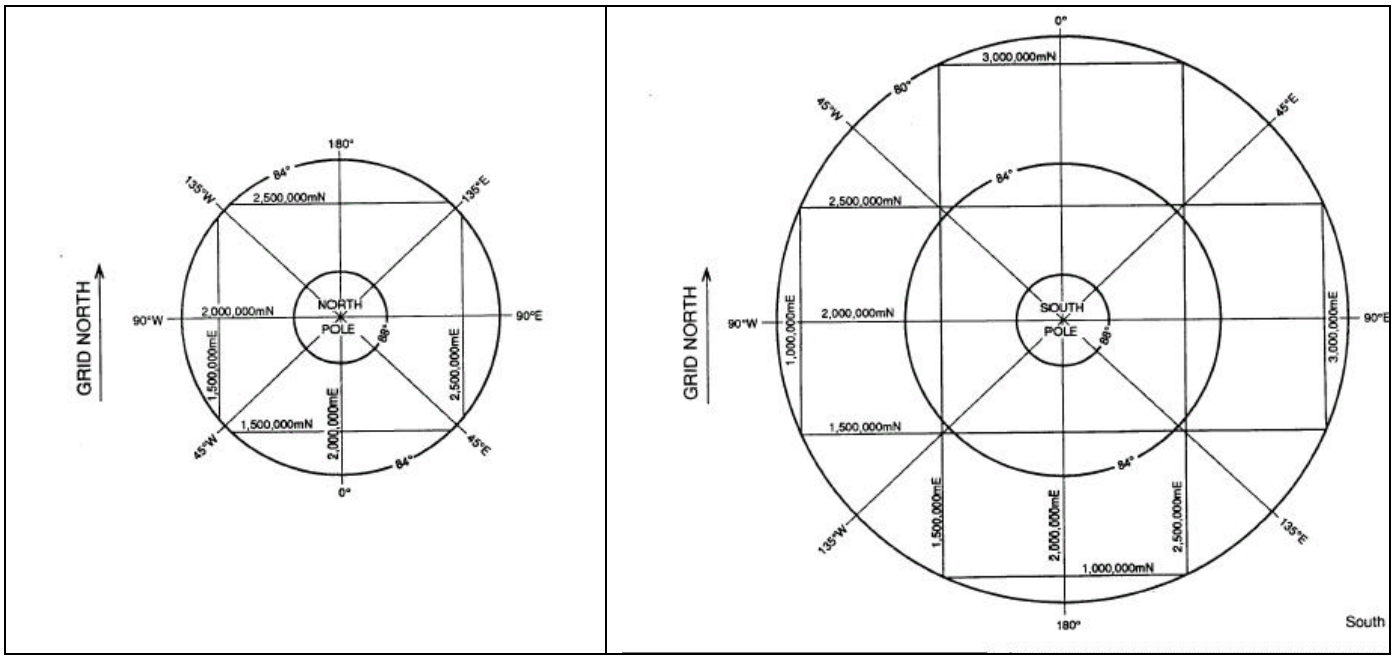


UPS - North Region
Universal Polar Stereographic

The above figure shows the diagram of the UPS projection in the northern polar region. As in all stereographic projections, the perspective point is at the point across the earth from the central point. Here this is the south pole. For the UPS southern projection the images in inverted and the north pole is the projection point.

As with the UTM, the coordinates are expressed in Easting and Northing meters. However the definition of these values differs from the UTM. In concept the are the same, meters from the center point. There is also **a false easting and northing** added. In this case they both are 2 million meters. Thus the coordinates of the pole are 2,000,000 N and 2,000,000 E.

The azimuth becomes less meaningful in polar regions. Therefore the grid north and grid azimuth are commonly used. These differ greatly from true azimuths on UPS maps. The reference line for the grid, in both hemispheres, is the line from the south pole to the north pole along the prime meridian.



The above diagrams show the coordinates in both polar regions to the same scale. Notice that there are nowhere negative numbers given this chose of offsets.

The formulas for the easting (x) and northing (y) are fairly simple for the polar case.

$$x = Rk\cos\phi\sin\lambda$$

$$y = \mp Rk\cos\phi$$

$$k = \frac{2k_0}{1 \pm \sin\phi}$$

where the upper sign is used in the northern hemisphere and the lower sign in the southern hemisphere. Of course, these become much more complex for the ellipsoidal model of the earth.

IV. Military Grid Reference System (MGRS)

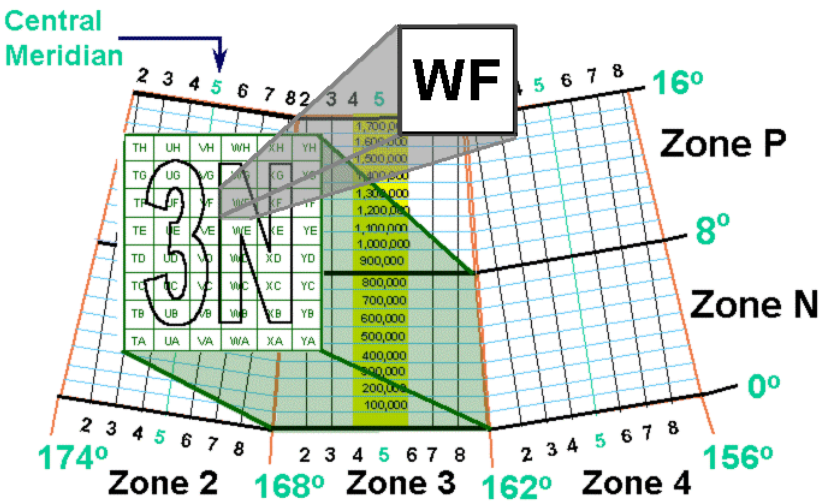
The Military Grid Reference System is just a re-labeling of the UTM (or UPS) easting and northings to avoid large numbers. It makes use of the fact that a map or operational area will usually only cover a small fraction of a UTM zone. It assigns letters to boxes in a hierarchical system. (It is easier and less error prone to send letters in Morse code, a common method of communication when this system was developed.) Only a summary of the system will be given here. A complete description is in reference 4.

The zones define the longitude belts or zones. They use the standard UTM zone numbers. The longitude zones are identified by numbers, the latitude zones by letters. The latitude is broken into 20 zones, 8 degrees tall except the last. These are lettered from south (80 S to 72 S) to north with the letters C to X omitting "O" and "I". These are given in the table below. Notice that the north most segment, X, is wider.

Latitude Limits	**MGRS Letter**	**Latitude Limits**	**MGRS Letter**
80 S to 72 S	C	0 to 8 N	N
72 S to 64 S	D	8 N to 16 N	P
64 S to 56 S	E	16 N to 24 N	Q
56 S to 48 S	F	24 N to 32 N	R
48 S to 40 S	G	32 N to 40 N	S
40 S to 32 S	H	40 N to 48 N	T
32 S to 24 S	J	48 N to 56 N	U
24 S to 16 S	K	56 N to 64 N	V
16 S to 8 S	L	64 N to 72 N	W
8 S to 0 N/S	M	72 N to 84 N	X
North Polar	Y and Z	South Polar	A and B

The Naval Postgraduate School, in Monterey CA is at about 36 35 N and 121 52 W. This puts it in zone 10. The above latitude nomenclature means that NPS is in MGRS location 10S. Of course this is a huge area. In order to further specify locations squares 100,000 m by 100,000 m (100 km by 100 km) are given letter two labels. The first letter specifies the east-west location and the second the north-south location. (This is in the same order as "x, y" used to plot the map.) The 100,000 meter square for this point is FF. There are 6 sets of 100 km box designators which cycle with the UTM zone numbers. This keeps the same Box Identification from being confusing if the UTM zone is left off.

At this point I need to point out that there are two MGRS's. The "old MGRS" and "new MGRS". The most important difference is the 100 km square letter assignment. The ellipsoid is also different. While most MGRS coordinates in use today should be the "new" version, there can be some confusion. In order to help, the square identification places the locations 1000 km apart for the same designation.



MGRS Zone System

It is common to truncate the MGRS locations to the accuracy needed. So the location of NPS would be reported with different number of values in different situations. Here is an example. The east value is first because this is the order for MGRS. Notice in the meter level MGRS that the numbers on the end are just the UTM coordinates leaving off the values beyond 100,000's. These are taken care of with the latitude code "S" and the 100,000 m square code "FF".

Coordinate Type	East-West Value / MGRS	North-South Value
Geographic	121 52 29 W	36 35 42 N
UTM Zone 10	600,653 E	4,050,535 N
MGRS to Meter	10SFF0065350535	
MGRS to 100 m	10SFF006505	

References

1. NIMA Geodesy and Geophysics Online Reference Material
http://www.nima.mil/GandG/pubs.html

2. USGS Map Information Page
http://mac.usgs.gov/mac/isb/pubs/pubslists/index.html

3. Snyder, John P., *Map Projections, A Working Manual*, US Geological Survey Professional Paper 1365 , 1987, US Government Printing Office.

4. *Datums, Ellipsoids, Grids, and Grid Reference Systems*. Edition 1. US National Imagery and Mapping Agency (NIMA). TM 8358.1, Washington, D.C., NIMA, 20 September 1990. (Available at reference 1 website as a PDF file.)

5. *The Universal Grids: Universal Transverse Mercator (UTM) and Universal Polar Stereographic (UPS).* Edition 1. US National Imagery and Mapping Agency (NIMA). TM 8358.2 Washington, D.C., NIMA, 18 September 1989. (Available at reference 1 website as a PDF file.)