Notes on Gravity

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I. Gravity, the Shape of the Earth, and Latitudes

All the stars, planets and larger moons are basically spheres. These bodies are shaped by the force of gravity that holds them together. The sphere is the shape that minimizes the energy of the system. This is the basic principle that shapes these bodies. However the force of gravity isn't the only force present. Also the mass distribution of a planet isn't precisely uniform.

The shape of the earth, and the coordinates we assign to positions, are significantly influenced by these differences of the real world from the simple spherical model. Here we will discuss three levels of approximation:

1. The simple spherical earth model
2. The "geodetic" model using an ellipse of revolution for a shape
3. The real world, which includes variations due to mountain etc.

The geodetic model, that based on an ellipse rotated about its shorter axis, is the common reference system used today. Variations from it are considered small changes or perturbations in the notation commonly used. Many of the complexities of the current descriptions come from defining quantities, like latitude, that agree (almost) with the measurements made in the past before some of the facts were known.

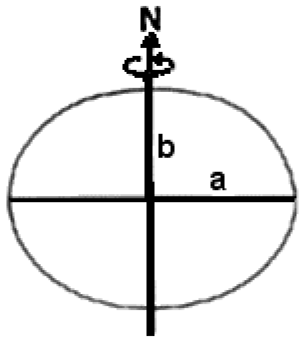
The gravity field of the earth is often described in high school and college physics books based simply on the force Newton discovered. This description would be correct for the real world, if it were a rigid sphere that did not rotate. The real earth rotates. The rotation imparts a "force"[†] on all things that rotate with the earth. This rotational effect is added to the normal gravitational attraction found in elementary physics books. In addition, this rotational effect causes the earth to assume a non-spherical shape.

The "common sense" definitions of gravity (derived from what we personally feel and see) were locked in place before the facts were known. Geodesy has adopted a scientific equivalent of these common sense definitions.

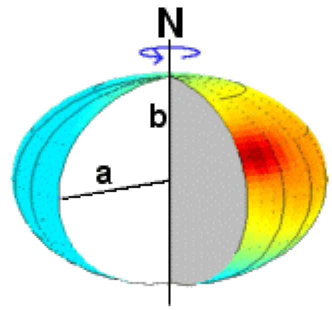
What we call "gravity" in geodesy is the sum of the **Newtonian gravitational attraction** and rotational effects. The rotation effect is called **centrifugal acceleration**. All things co-rotating with the earth feel it. This causes the earth to have a bulge at the equator. For a uniform earth, the shape is an ellipsoid of revolution. (This is also called a spheroid). This is the geodetic

[†] This is often called a pseudo-force. It is felt only by objects rotating with the earth. It arises from the non-inertial nature of the earth fixed rotating coordinate system. Satellites are not fixed to the earth and do not feel this force.

model we will discuss in this section. The complexity of the bumps in the real world will be discussed in the later sections.



Rotate Ellipse



Making Ellipsoid

One of the ways to define the amount the ellipse differs from a circle is called the **flattening**, f. The flattening, f, is given by the equation

$$f = \frac{a-b}{a} = 1-\frac{b}{a} \,,$$

where a is the longer equatorial radius and b is the shorter polar radius. The flattening of the earth is about 1/300. If the earth were a fluid it would be about 1/230. The earth acts almost as a fluid over long time frames. Newton was the first to predict the flattening of the earth. Another common way to describe the shape of the ellipse is with the eccentricity, e. This is defined as:

$$e = \sqrt{1-\frac{a^2}{b^2}} \,, \text{ or}$$

$$e^2 = 1-\frac{a^2}{b^2} \,.$$

The observed acceleration of gravity is the sum of the Newtonian gravitational attraction and the rotational force. The shape of the earth minimizes the potential energy due to the two forces. This makes the direction of the observed acceleration of gravity perpendicular to the ellipsoid. This line does not go to the center of the earth except for the poles and points on the equator.

The Newtonian accelerations is given by

$$\vec{g}_n = -\frac{GM}{r^2}\hat{e}_r$$

where

G is the universal gravitational constant, M is the mass of the earth,

r is the radius of the earth (or distance from earth center of mass to observer), and

$\hat{e}_r$ is a unit vector that points outward from the center of the earth to the observation point.

This equation is only valid outside the earth. Inside the earth only the below the observer contributes to the Newtonian acceleration.

The centrifugal acceleration is given by

$$\vec{a}_c = \omega^2 p \hat{e}_p$$

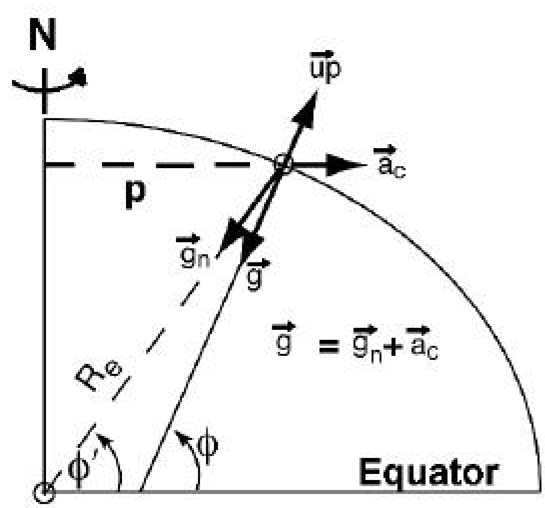
where

$\omega = 2\pi / T$ is the rotation rate of the earth (T is 1 sidereal day),

p is the distance from the axis of rotation to the observer shown in the figure, and

$\hat{e}_p$ is a unit vector along the outward direction of p.

This applies only to objects that rotate with the earth. Satellites in space do not feel this acceleration.



Quarter of Earth
Ellipsoid Crossection

The Newtonian gravitational attraction, which we will denote g_n, does point at the center of mass of the earth. But the sum of forces we feel does not. The rotational acceleration, which we will call a_c, acts outward from the axis of rotation. These accelerations are shown in the diagram. Note that the vectors are not drawn to scale. (**Geodetic (ϕ) and geocentric (ϕ') latitude** are also shown on this diagram.)

The observed acceleration, g, is defined by the vector sum of the g_n and a_c . The total is the earth's acceleration of "gravity". The rotational acceleration term is maximum at the equator and zero at the poles because the moment arm p varies with latitude. In the real world, the maximum a_c is smaller than g_n by a factor of 300. Therefore the effects on g, and angles, are small. But the earth is large and even small effects on angles can cause changes of 10's of km.

The nominal Newtonian acceleration is about 980 cm/s^2 and the maximum rotational term about 3 cm/s^2. In geodesy accelerations of gravity are called **gal**'s, short for Galileo. <u>One gal is one cm/s^2</u>. This is a large unit for measuring variations, so the milligal, mgal or 1/1000th gal, commonly occurs.

The gravity value at each geodetic latitude for an ellipsoid is given by:

$$g \quad = \quad \frac{g_e \cos^2\phi + g_p\sqrt{1-e^2}\sin^2\phi}{\sqrt{1-e^2\sin^2\phi}}$$

where g_e is the acceleration at the equator, g_p, is the acceleration at the poles, and e is the eccentricity. This is not as complicated as it looks. It is easy to show that

$$\frac{b}{a} = \sqrt{1-e^2} \quad .$$

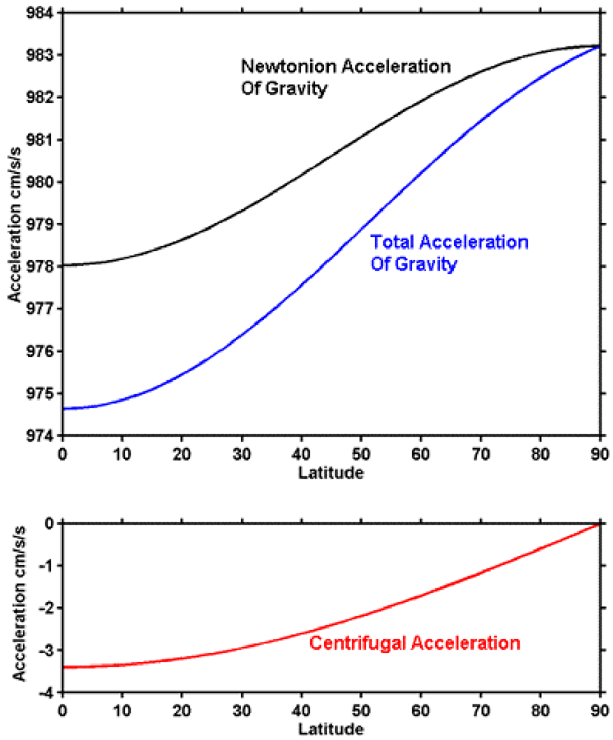
This equation is often written using the symbol gamma, γ, instead of g. Often gamma is used for theoretical gravity from a perfect ellipsoid and g for the real gravity from the lumpy, non-uniform real earth. Another form of this equation is

$$g = g_e \frac{1+k\sin^2\phi}{\sqrt{1+e^2\sin^2\phi}} ,$$

with the auxiliary value k given by

$$k \quad = \quad \frac{b}{a}\frac{g_p}{g_e} \quad -1.$$

This value is graphed in the next figure along with the two components.



Components of Gravity Acceleration

Total is vector sum of components

Newtonian Acceleration -- Toward Center of Earth
Centrifugal Acceleration -- Outward From Polar Axis
Total Acceleration - Perpendicular to Ellipsoid

The two components and the total acceleration of gravity on the ellipsoid are a function of latitude as show in the above figure. The total acceleration is obtained by adding the Newtonian term and the centrifugal term as vectors. The Newtonian acceleration is opposed by the rotation terms everywhere except at the poles. It is therefore larger than the total value. The Newtonian term increases at the poles because the surface is closer to the center of mass. The gravity at the poles is about 5 gal (5 cm/s/s) larger than at the equator. About half of this effect is being closer to the center and the rest is the difference in the rotation term, the centrifugal acceleration.

The direction "Up" is also defined from the sum of the forces. It is perpendicular to the ellipsoid surface. It's inverse, "Down", does not point to the center of the earth. We historically defined latitude from observations of stars, with respect to the local up called the **local vertical**. The historical definition of latitude corresponds to the angle made the line perpendicular to the ellipsoid, not the one to the center of the earth. The local vertical is perpendicular to the ellipsoid in the absence of inhomogenates.

The latitude we see on maps, called **geodetic latitude**, comes from this perpendicular to the ellipsoid. This latitude is commonly called geographic latitude, but this term is not well defined in the scientific literature. Officially it is geodetic latitude. Usually when you see a term called geodetic something, it refers to definitions based on the ellipsoid. The coordinates using the vertical sensed by a bubble level, which responds to the real variation in the gravity field, are discussed in section VII.

The angle made by the line to the center of the earth is called **geocentric latitude**. This is the only latitude in the spherical earth model. Today geocentric latitude is used mainly in the field of artificial earth satellites.

II. Terminology – Geodesy vs. Physics

In the field of Geodesy, there are some terms and concepts that "look and feel" like things studied in elementary physics, but are slightly different. This can cause some confusion, especially if one takes equations from both physics books and geodesy books.

	Physics	**Geodesy**
Gravity	Newtonian Gravitation	Newtonian Gravitation Plus Rotational Effects
Force	Force, F=ma Body Force f= F/m	Force per unit mass, Acceleration, g, As seen on Rotating Earth
Potential	Force = - Gradient (Potential) $\vec{F} = -\vec{\nabla}V$	Acceleration = + Gradient (Potential) $\vec{g} = +\vec{\nabla}V$
Coordinates Polar Angle	Measured from Pole (0 to 180 degrees)	Measured from Equator (ϕ') (-90 to 90 degrees)

The main source of difference is the inclusion of the rotational effects in the terms used in geodesy. This is done to reflect the measurements seen by an observer rotating on the earth's surface, that is measurements made on the real world. The definitions of several items were made before many of the concepts used in today's elementary physics were discovered. Newton or his contemporaries made many definitions. The concepts of potential energy, kinetic energy, and the conservation of energy followed this work by 100 years.

The second difference is the relation between the force or acceleration and a **potential function**. First geodesy deals in accelerations, not forces. This is because the effects of the mass of an object in Newton's second law (F = ma) and his law of gravity ($F = GMm/r^2$) cancel out.

<u>In geodesy the acceleration is taken as the</u> **gradient**‡ <u>of the potential.</u> In physics the force is the negative gradient of the potential energy. In geodesy the negative sign is omitted. In physics terms, the potential energy of something sitting on the earth is negative. You have to add energy to move it off the earth. In geodesy the gravity potential is a positive number. It is the absolute value of the physics value of potential energy per unit mass. The two potentials are related by a minus sign.

‡ The gradient is a measure of how fast a function changes. In this case the changes are with respect to position, a rate of change with distance. The gradient is a derivative. It is also a vector. It points in the direction of maximum positive change. For a spherical earth, the gradient of the gravity potential points to the center of the earth. For the real earth it points in the local down direction. (Inverse of the local vertical).

III Potential and Gravity Acceleration

It is important to distinguish **accelerations** from **potentials**. What we feel is the acceleration. What are usually computed are the potential surfaces. Not all forces can be derived from potentials, but those involved in gravity can.

A potential is usually a **scalar** function of position. It is a number at each location. The force or acceleration is a vector value at these locations. It has magnitude and direction and has three components. The force or acceleration is derived from the potential by taking a rate of change (derivative) with respect to distance. The gradient operator is used. The force, or acceleration, is a function of how close surfaces of constant potential are to each other - the gradient. Close spacing means a large gradient and more acceleration.

In physics a force is the negative gradient of the potential. Things tend to move to the lowest potential energy. A similar relationship exists for the potential in geodesy. Here the acceleration (force per unit mass) is the positive gradient of the potential.

A potential can be written for each of the two components of gravity. These equations will be in the geodesy notation. The Newtonian part of the gravity field has a potential usually called V,

$$V = \frac{GM}{r} \ .$$

In physics the same symbol is used for the potential energy, but has the opposite sign.
The acceleration force from this, in the geodesy notation, is,

$$\begin{aligned} \vec{g} &= \vec{\nabla} V \\ &= -\frac{GM}{r^2}\hat{e}_r \end{aligned}$$

Notice that this final equation is the same as found in physics books. The two sign differences between physics and geodesy notation have canceled. This is always true when you relate physical things like mass or density and accelerations.

A potential function can also be written for the **centrifugal acceleration** part of gravity,

$$\Phi = \frac{\omega^2 p^2}{2}$$

with

$$\vec{a}_c = \vec{\nabla}\Phi \ .$$

Here p is the moment arm, the distance from the polar axis to the surface parallel to the equator. (If we use a Cartesian coordinate system with a its origin at the center of the earth and z being the polar axis, then $p = \sqrt{x^2 + y^2}$.) This value is shown in the diagram of the ellipse above.

The **total potential** is usually called W and given by

$$W = V + \Phi \ .$$

The observed gravity acceleration is given by

$$\vec{g} \quad = \quad \vec{\nabla} W \ .$$

What we normally call the acceleration of gravity is the magnitude of $\vec{g}$. This is the value that determines the period of a pendulum clock.

IV. Potential Surfaces, Geoid, and Heights

The surfaces of constant W are surfaces of constant potential in the geodesy nomenclature. These are also surfaces of constant potential energy. The minus sign doesn't change the form of a constant surface. Water would assume one of these surfaces in a bowl, lake or ocean. These surfaces are called "**Level Surfaces**"[§]. **Mean sea level** is one particular surface of constant W. This particular surface can be extended over land. This particular level surface is called the **geoid**.

To this point, the earth has been idealized. But the real earth has oceans, mountains and other density variations. These variations are small, but cause the real gravity values to differ from the simple ellipsoid model. In the ellipsoid model, the geoid is an ellipsoid. In the real world the geoid has bumps. Now we will turn to the effects of the inhomogenates of the real world. The largest effects will be on heights.

The form of the surfaces of W must be measured; they cannot be computed from a few constants and a model. They depend on the real world mass variations, which varies on many distance scales. Computing the distance from the geoid to the center of the earth is not possible without large numbers of real world measurements. The ellipsoid is defined mathematically with respect to the center of the earth and its radius is easily computed. Knowing the distance between the ellipsoid and geoid therefore would define the geoid. The form of the geoid over large distances was not well known until satellites were launched.

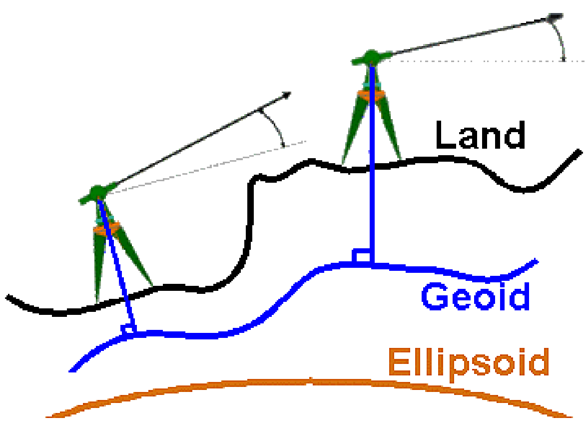
Starting with a "stake in the sand" that defines mean sea level at the shore, the heights of the land can be measured. With classical survey techniques, this is done with a series of measurements made from point to point.

These heights measured by classical methods are measured from the geoid. This happens because of the way heights are measured. Transits (telescopes on a tripod) are used, with angles measured with respect to the **local vertical**. The local up vector measured with a **plumb bob** or **spirit level** is the perpendicular to the real geoid.

[§] Level Surfaces are the three-dimensional analogue of contours in two dimensions. In one dimension if you find the locations where the function has a particular value, you get a set of points. If you have a function of two dimensions, such as height as a function of latitude and longitude, the set of points of a particular value is a line. This is one of the contours. If you have a function of latitude, longitude and height the set of points of a particular value is a surface. Picking one potential defines a surface. One particular value defines the geoid.

1908 Transit Theodolite

As the transit is moved from place to place, the local vertical will follow the hills and valleys of W. Therefore the geoid is the reference surface for classical height measurements. As one chief of the US National Geodetic Survey once said, “we are in the position of knowing heights very accurately with respect to a surface we know poorly”.



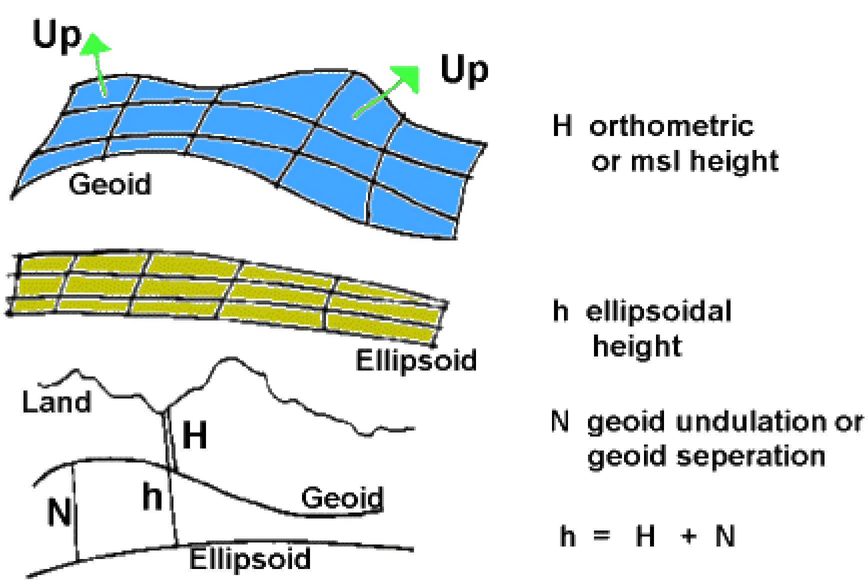
Classical Surveying
Follows Geoid

Heights determined in this manor are called **orthometric heights** or **mean sea level (msl)** heights. (Orthometric and msl height are slightly different, but the two terms are commonly used to mean the same thing.)

Heights determined in this manor are called **orthometric heights** or **mean sea level (msl)** heights. Except in geodetic science literature, where you see msl height, it usually means orthometric height.

Technically orthometric and msl height are slightly different. Ocean currents and other effects slightly modify the real surface height of the sea. All the difference between real msl and orthometric heights can be classified as "oceanography".

Heights measured from the ellipsoid are called **ellipsoidal heights** or **geodetic heights**. Satellite systems measure ellipsoidal heights. This is not what we see on maps or in databases. These contain orthometric height. Because satellite based positioning is now very important, there is a need for a good measurement of the geoid location. This is done by measuring the vertical distance between the ellipsoid and the real world geoid as a function of location. This difference is called **undulation of the vertical** or **separation of the geoid**. The symbol N is used for the undulation.



Geoid - Ellisoid Diagram
Two and Three Dimensions

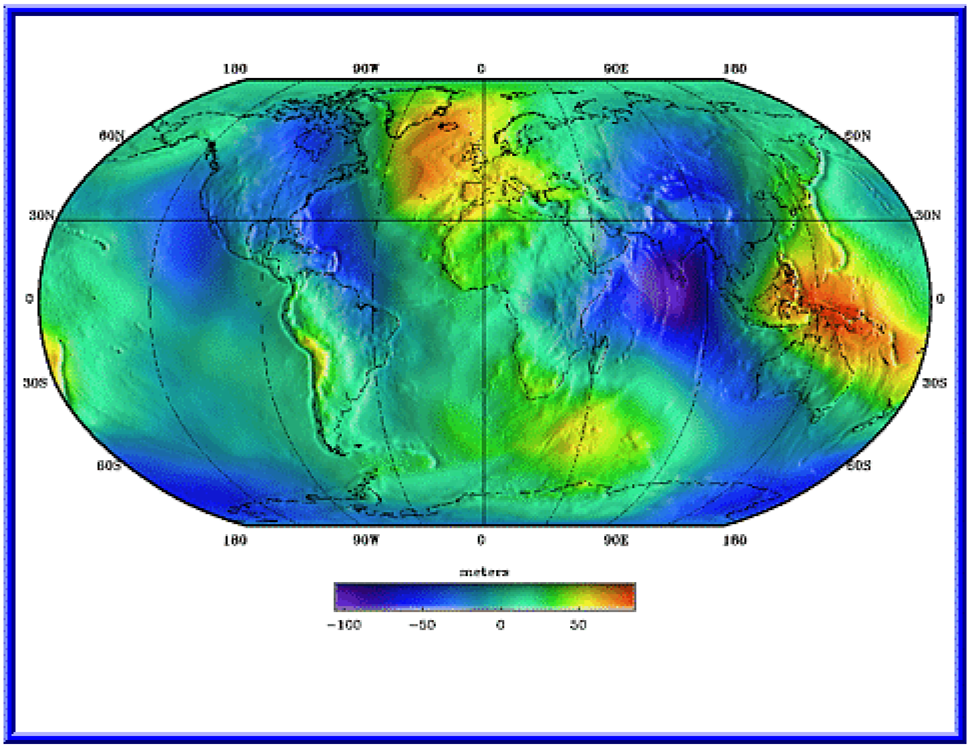
While the symbols h and H are commonly used for the two heights, there is no standard for which means orthometric or ellipsoidal. Here we will use the DoD standard,

h for ellipsoidal, and
H for orthometric height.

Note that in all cases the undulation, N is the orthometric minus the ellipsoidal heights. It is the distance from the ellipsoid to the geoid.
Thus

$$\begin{aligned} h &= H + N \\ N &= H - h \end{aligned},$$

where the values are taken at the same point. N is a function of location. Finding N as a function of latitude and longitude is how we determine the geoid.



The above figure gives a worldwide view of the geoid surface. The values range from about -100 m to + 100 m. World wide ellipsoids are chosen so that the average undulation is zero over the world. This means that there are as many places with the geoid above the ellipsoid as there are with the geoid below the ellipsoid. There are many small area variations in the geoid that do not show up in this figure.

Because orthometric heights (msl) were the only ones historically measurable, they are the heights shown on maps and in databases. All heights on maps are orthometric. However position measurements made using satellite systems, such as GPS, are inherently done in an earth centered, earth fixed Cartesian X-Y-Z system. These can easily be converted to latitude, longitude, and ellipsoidal height. (Also called geodetic height). The geoid undulation, N, is needed to convert these to map type (orthometric or msl) heights. This value is only know at the meter level in most places.

V. Potential and Gravity Variations from Nominal

In order to deal with small variations, a model is used to subtract out the main, large effects. If you put all the mass of the earth into the ellipsoid, then you can compute the gravity field. This theoretical potential is called U. The size of the ellipsoid is chosen such that the value of U on the ellipsoid is the same as the true potential value of W on the msl surface.

There are pairs of things. We have the theoretical values from the perfect ellipsoid and the real values from the real inhomogeneous earth. The theoretical potential function that goes with the model ellipsoid is called U. This corresponds to the real potential W. The gravity value for the model is called $\vec{\gamma}$, which corresponds to the real value $\vec{g}$.

The model acceleration, γ, is found from U with the gradient just as the real gravitational acceleration is found from W,

$$\vec{\gamma} = \vec{\nabla} U$$

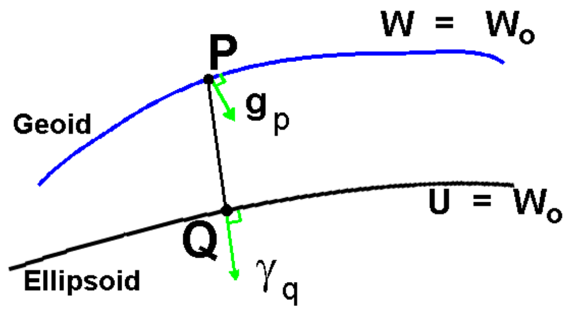
The nominal or "normal" acceleration is ã, the magnitude. The value of γ is only a function of latitude. The real gravity acceleration is a function of latitude and longitude.

We now define some differences, or perturbations. The difference in potential,

$$T = W - U .$$

is called the **disturbing potential**. In the case of the acceleration, there are two differences. The one found most useful is called the gravity anomaly. The gravity disturbance is more intuitive, but seldom used due to practical difficulties.

The **gravity anomaly** was defined so that it could be measured without knowing the **geoid undulation, N**. This was important prior to accurate knowledge of N.



$$\Delta g = g_p - \gamma_q$$

Gravity Anomaly

The gravity anomaly is the difference of the measured gravity acceleration, "reduced to the geoid", and the theoretical gravity on the ellipsoid,

$$\Delta g \quad = \quad g_P - \gamma_Q$$

where Q is a point on the ellipsoid and P is the point "above" it on the geoid. Reduction to the geoid is done with measured heights (orthometric heights) and an iterative process to correct for local masses. Thus one uses a pendulum or other device that measure the magnitude of the acceleration of gravity along with a measured orthometric height to get g_P. This is difference using the magnitudes of the vectors.

There are many different types of gravity anomalies with names like Bouguer, Bouguer Free Air, Helmert, Pratt-Hayford, etc. The real measurements are taken on the surface of the earth. To compute a gravity anomaly the value that would have be measured on the geoid is needed. The different types of anomaly use different methods to adjust the measurements to what would have been measured on the geoid.

The gravity disturbance is the difference with both points on the geoid. The problem is that the undulation, N, is needed. This was generally not well known before satellites.

VI. Brun's Theorem and Stokes' Integral

The disturbing potential can be directly related to N via **Brun's theorem**. Consider going by boat from Monterey Ca, to Santa Cruz across Monterey Bay. It is know that the geoid is 1 meter higher at Santa Cruz. Staying on the ocean, you are at the same real potential, the same W. But you are 1 meter higher with respect to the ellipsoid. The potential energy difference is the acceleration of gravity times the height moved. Therefore U is 1m * γ larger. If W is constant, and U changes, then T = W – U must change also. So any difference in N must be accompanied by a corresponding change in T. Brun's theorem is therefore,

$$N = \frac{T}{\gamma_0}$$

Here ã$_0$, the normal gravity on the ellipsoid, is used by convention.

In the mid 1800's Stokes found a mathematical relationship between the gravity anomalies and the undulations. The undulations are given by the integral of the anomalies times a function (called the Stokes function, s(ψ))

$$N \quad = \quad \frac{R}{4\pi\gamma} \oint\!\!\oint_{\text{Earth Surface}} \Delta g \; s(\psi) \; dS \; .$$

The critical feature of this equation is that the gravity anomalies, Δg, are needed over the entire earth. There are many techniques that were used to make up for the lack of anomalies. They all resulted in recovering the local, small-scale variations in N. However these values always were biased. Only the form of the local hills and valleys were determined. This was enough for some applications such as oil prospecting.

VII. Deflection of Vertical and Astrodetic Coordinates

The **"up" vector, or vertical**, is along the line of the gradient of W, the true potential. The gradient of U will give the perpendicular to the ellipsoid - the official definition of up. Because W is not quite U, there will be a difference. This small difference in theoretical and measured verticals is called the **deflection of the vertical**. It is usually under an arc minute, often only a few arc seconds. It has components in the north-south and east-west.

The up vector responds to minor bumps in the geoid. The undulation can vary only a small amount, while the normal can vary quite a bit. The defection is more sensitive to near by density variations (mountains, etc.) than the undulation.

The deflection of the vertical will normally be in some general direction, not north-south or east-west. Therefore it has two components. The north south (latitude) component is usually called ξ (Greek Xi) and the east west η (Greek Eta). Therefore the deflection of the vertical has two components. These define the difference between geodetic and **astrodetic (or astronomic) coordinates.**

The **astrodetic latitude** and **longitude** are measured using the local vertical. They are related to the geodetic latitude and longitude (which use the perpendicular to the ellipsoid) by,

$$\sin(\phi) = \cos(\eta)\sin(\Phi - \xi)$$
$$\sin(\eta) = \cos(\phi)\sin(\Lambda - \lambda)$$

Where the capital Phi and Lambda are astrodetic values and the small letters are geodetic. Because the deflection is small, $\cos(\eta)$ can be taken as 1 and $\sin(\xi)$ as ξ. Then

$$\xi = (\Phi - \phi),$$
$$\eta = (\Lambda - \lambda)\cos(\phi)$$

Definition and Notes for Some Terms

Centrifugal acceleration

The acceleration outward (centrifugal acceleration) from the axis of rotation. Caused by being attached to rotating with, the earth.

Acceleration of gravity

The total acceleration felt on the earth. Usually means the scalar value, not the vector. Has components due to the Newtonian gravity and centrifugal acceleration.

Newtonian gravity

Attraction of two masses for each other. Published by Newton in 1672.

Newtonian potential

The potential function for Newtonian gravity.

Rotational potential

A potential function that produces the centrifugal acceleration.

Gravity Potential, W, U

The total potential, the sum of the Newtonian and rotational part. When given for a model earth, and ellipsoid, the value is called U. When given for the real earth it is called W.

Ellipsoid

A model of the earth formed by rotating an ellipse about its shorter axis. This is the polar axis. For modern models, world wide data is used. The center is the center of mass of the earth. (This is sometimes called the barycenter.) The size is set so the average Undulation of the vertical is zero. Models earlier than WGS72 used data from a restricted area of the world for a best fit of that region. For modern world geodetic systems, the potential U on the ellipsoid is the same value as the real potential on the geoid. It is the reference for ellipsoidal heights (sometimes called geodetic heights). Satellites measure ellipsoidal heights. The ellipsoidal height is called both "h" and "H" in the literature. Current US DoD nomenclature is to call it "h".

Geoid

A level surface. This is a surface of constant W. Water would settle on this surface if the land did not get in the way. It is often called the surface of mean sea level. It is continued under the land. It is the reference surface for all heights found by classical surveying (orthometric heights). These are often called msl heights. This is the type of height reported on maps. The orthometric height is called both "h" and "H" in the literature. Current US DoD nominclature is to call it "H".

Undulation, Undulation of the Vertical, Separation of Geoid

This is the distance measured from the ellipsoid to the geoid, perpendicular to the ellipsoid. Is is usually called N. In the above nomenclature, $N = h - H$. It can be either positive or negative. It is generally in the range of +/- 200 m.

Gravity anomaly

This is the historically used difference between the observed and theoretical acceleration (scalar) of gravity. The observed value is "reduced" to the geoid. The theoretical value is taken on the ellipsoid. It is the input to Stokes integrals used to convert gravity measurements to undulations.

Gravity disturbance

This is the difference of the observed and theoretical acceleration of gravity with both values on the geoid. Moving the theoretical value to the geoid requires knowledge of the undulation. It is accurately found only in modern times when good models of N are available.

Deflection of vertical

The difference between the vertical measured from local gravity and the perpendicular to the ellipsoid. It is the basis of the difference between geodetic and astrodetic coordinates. It is normally only a few arc seconds.

Orthometric height

Heights measured from the geoid. The type of height on maps. Often called mean sea level height. The kind of height produced with classical surveying.

Ellipsoidal height

Heights measured from the ellipsoid. The type of height measured by satellite systems. The undulation is needed to convert these to "map" heights.

Brun's Theorem

Relates the disturbing potential, T, to the undulation N. Obtained from conservation of energy arguments. To first order in small differences, $N = T/g$
or $N = T/\gamma$.